Parametrix approximations in option pricing

F. Corielli, P. Foschi and A. Pascucci

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– *Workshop Modelling and numerical techniques in quantitative finance* –

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- **Aït-Sahalia** (2002): Hermite polynomials approximation

Option pricing and fundamental solutions

A simple example: local volatility

$$dS_t = \sigma(t, S_t)dW_t$$

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Γ = **transition density of S / fundamental solution**

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First idea:

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- $\Pi(t,S;T,\xi)$ is a **Gaussian function in** (t,S)

The backward parametrix

$\Pi(z;\zeta)$ is a Gaussian function **as a function of** z but

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$$\Gamma(z;\zeta) \;=\; P(z;\zeta) \;+\; \text{“correction term”}$$

$P(z;\zeta)$ is a **Gaussian function in ζ**

The backward parametrix, II

Second idea: look for Γ in the form

$$\Gamma(z;\zeta) = P(z;\zeta) + \int_t^T \int_{\mathbb{R}} P(z;\cdot)W(\cdot\,;\zeta)$$

here $z = (t, S)$ and $\zeta = (T, \xi)$

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- recursive formula

$$W(z;\zeta) = LP(z;\zeta) + \int_t^T \int_{\mathbb{R}} LP(z;\cdot)LP(\cdot\,;\zeta) + \dots$$

Global error estimates

$$|\Gamma(t,x) - P_n(t,x)| \leq \delta \, \frac{t^n}{n!} \, \Gamma_{\text{heat}}(t,x)$$

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- rate of convergence independent on dimension
- asymptotically exact as $t \to 0^+$ and $|x| \to \infty$

Option price expansion

C option price with payoff ϕ:

$$C(t,S) = \int_{\mathbb{R}} \phi(\xi)\Gamma(t,S;T,\xi)d\xi = \sum_{n=1}^{\infty} C_n(t,S)$$

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- First term: **Black&Scholes price with $\sigma = \sigma(t,S)$**

$$C_1(t,S) = \int_{\mathbb{R}} \phi(\xi)P(t,S;T,\xi)d\xi$$

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- n-th term:
  B&S price with $\sigma = \sigma(t,S)$ and transaction cost LC_{n-1}

$$C_n(t,S) = \int_t^T \int_{\mathbb{R}} LC_{n-1}(\zeta)P(t,S;\zeta)d\zeta$$

Example 1: explicit 2-parametrix for local volatility

Pricing operator: $LC = a(\cdot)\partial_{SS}C + \partial_t C$

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Call option with strike K:

$$\boxed{C(t,S) \simeq C_{\mathrm{BS}}(t,S) + \frac{K(T-t)}{2}\left(a(T,K) - a(t,S)\right)P(t,S;T,K)}$$

Numerical test: CEV model

$$\frac{dS_t}{S_t} = r dt + \sigma_0 S_t^{-\alpha} dW_t, \qquad \alpha \in]0, 1[$$

- Black - Scholes for $\alpha = 0$

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- Matched Asympt. Expansion (MAE): **Howison** (2005)
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- maturity from 1 week to 1 year
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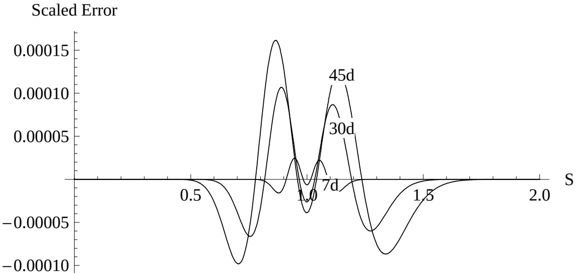
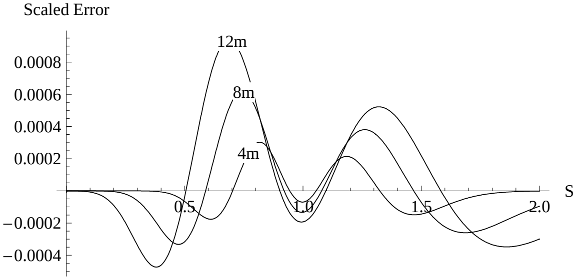
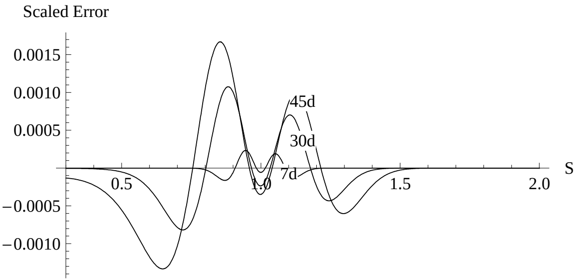
- **Cox** and **Shaw** approx. are equivalent
- **Hagan-Woodward** very accurate and *closed-form* solution $\Longrightarrow$ used as reference prices

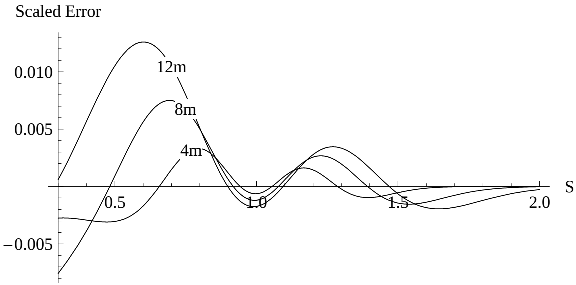
CEV: parametrix vs Hagan-Woodward, $\alpha = \frac{1}{4}$



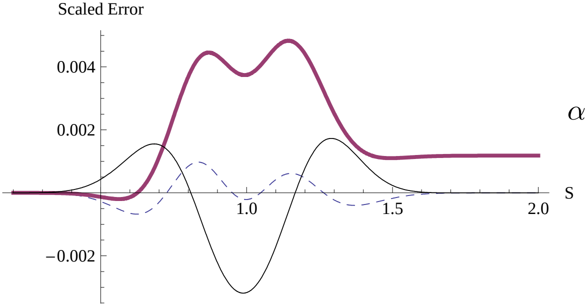


CEV: parametrix vs Hagan-Woodward, $\alpha = \frac{3}{4}$



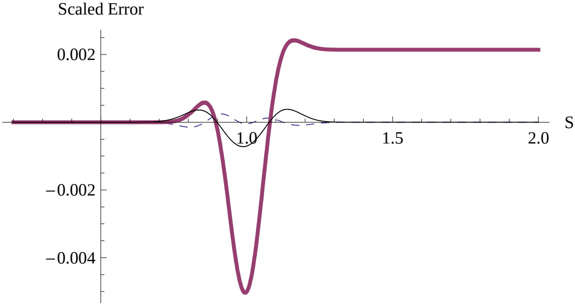


CEV: param. (dashed), MAE (thick), Svoboda-G. (black)



$\alpha = \frac{1}{2}$, $\sigma = 30\%$, $T = \frac{3}{12}$

$\alpha = \frac{1}{2}$, $\sigma = 15\%$, $T = \frac{3}{12}$



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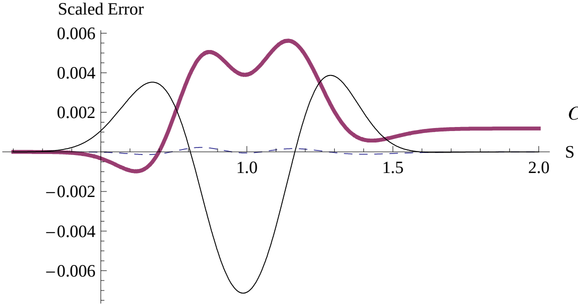

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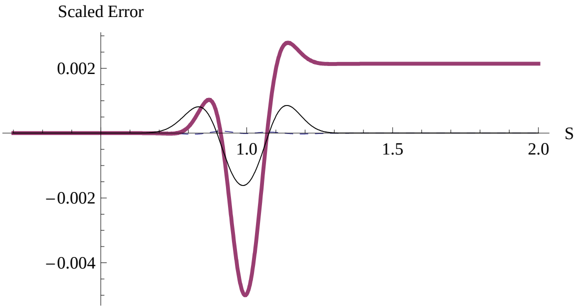

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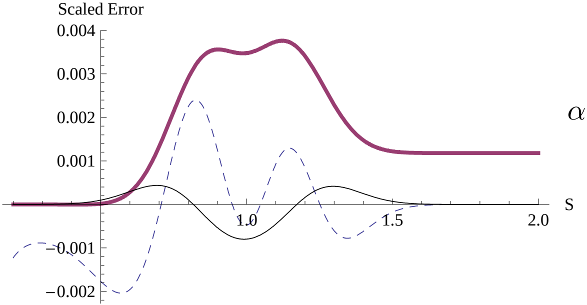

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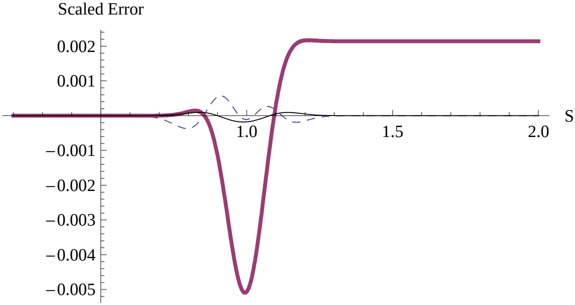
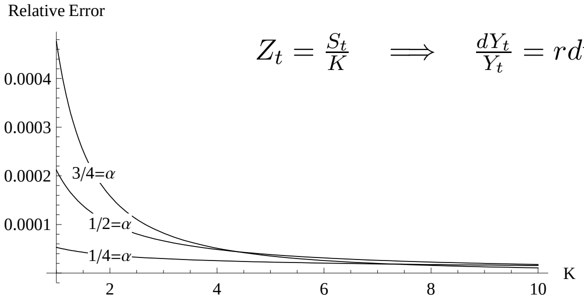
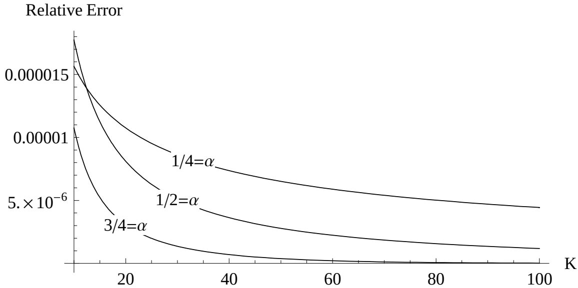


CEV: strike K large, volatility σ small

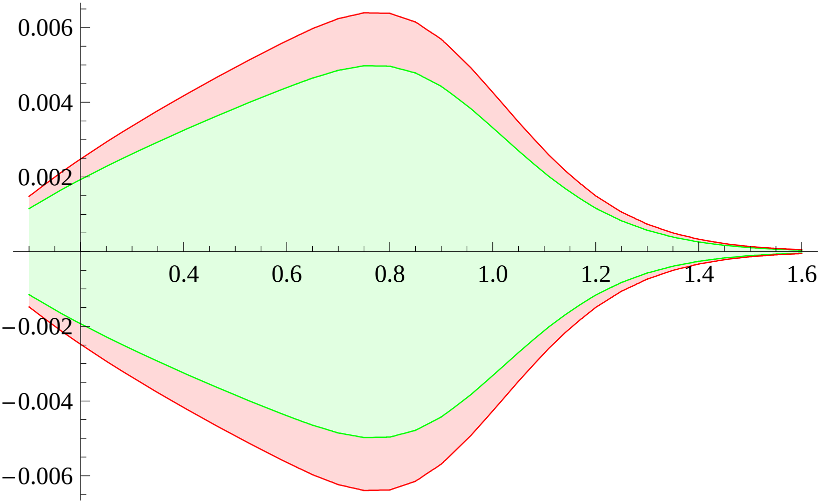
$$Z_t = \frac{S_t}{K} \quad \Longrightarrow \quad \frac{dY_t}{Y_t} = r\,dt + \frac{\sigma}{K^{\alpha}} Y_t^{-\alpha} dW_t$$





Monte Carlo

100-Euler, 500.000-MC, $\alpha = \frac{1}{2}$, $T = \frac{3}{12}$



Monte Carlo, Hagan (black)

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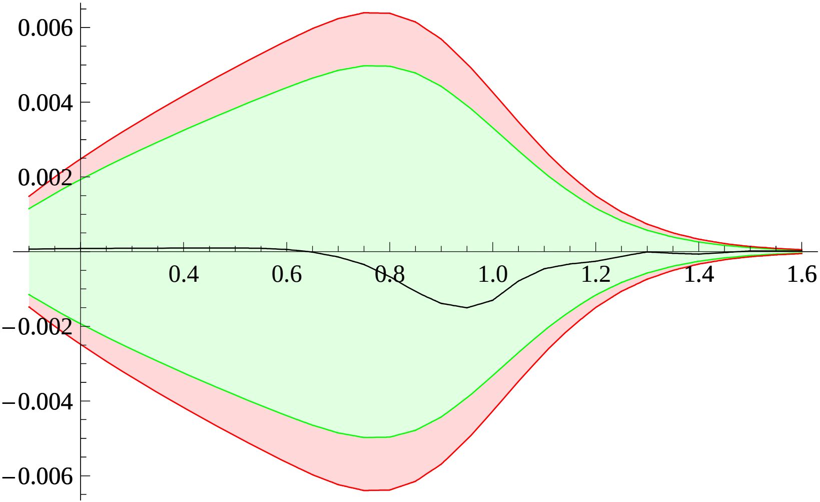
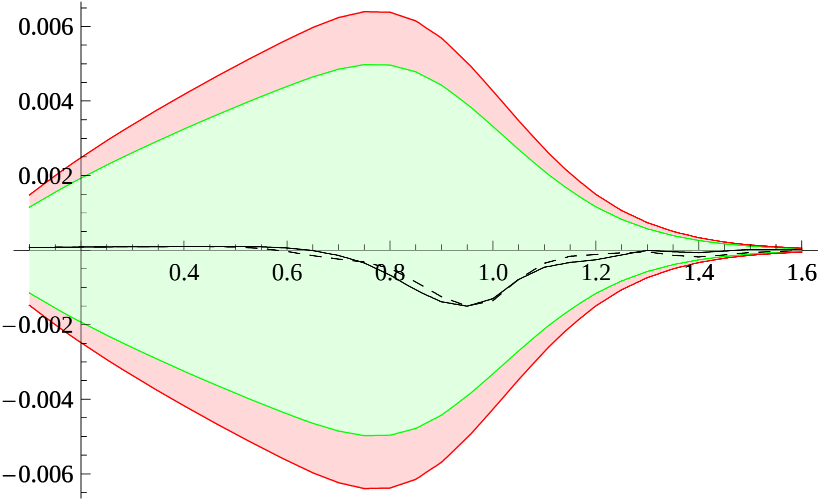


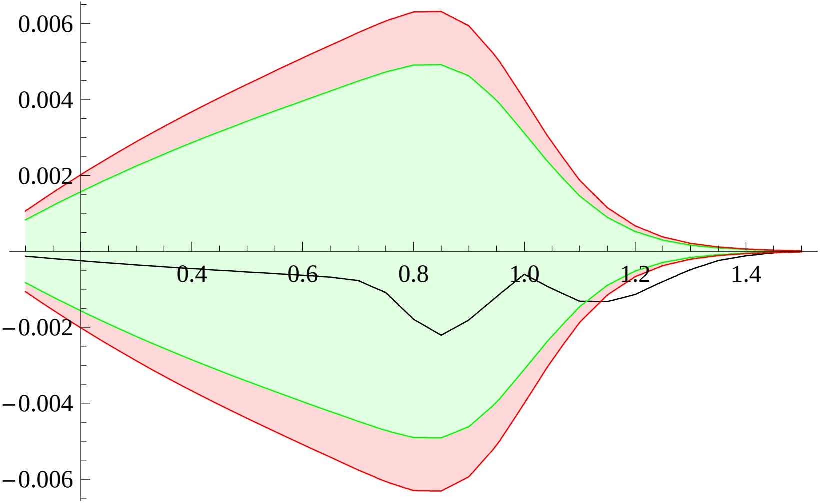
Monte Carlo, Hagan (black) and parametrix (dashed)

100-Euler, 500.000-MC, $\alpha = \frac{1}{2}$, $T = \frac{3}{12}$



Quadratic LV: $\sigma(S) = \sigma_0 \min\{2, \sqrt{1 + (S-1)^2}\}$

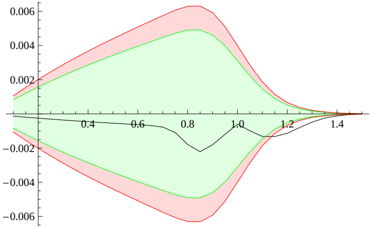
100-Euler, 500.000-MC, $\sigma_0 = 20\%$, $T = \frac{3}{12}$



Quadratic LV: MC vs parametrix

	MC	Parametrix
$S = 1$	0.046159	0.046149
$S = 1.1$	0.119926	0.119883
$S = 1.2$	0.213524	0.213482
$S = 1.3$	0.312541	0.312524
$S = 1.4$	0.412432	0.412429
$S = 1.5$	0.512423	0.512423
$S = 1.6$	0.612422	0.612422

- $\sigma(S) = \sigma_0 \min\{2, \sqrt{1 + (S-1)^2}\}$
- strike $K = 1$, maturity $T = \frac{3}{12}$
- 100-Euler, 500.000-MC



Path dependent volatility (2-dim)

Hobson-Rogers (1998), **Foschi-P.**(2007)

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Z_t = log-price

$$dZ_t = \mu(D_t)dt + \nu(D_t)dW_t$$

D_t = deviation from the normal trend

$$D_t = Z_t - \int_0^{+\infty} e^{-s} \, Z_{t-s} \, ds$$

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$$D_t = Z_t - \int_0^{+\infty} e^{-s}\ Z_{t-s}\ ds$$

Pricing PDE (degenerate parabolic):

$$a(x,y,t)\partial_{xx} + x\partial_y + \partial_t, \qquad (x,y,t) \in \mathbb{R}^3$$

Path dependent volatility (2-dim)

Hobson-Rogers (1998), **Foschi-P.**(2007)

Z_t = log-price

$$dZ_t = \mu(D_t)dt + \nu(D_t)dW_t$$

D_t = deviation from the normal trend

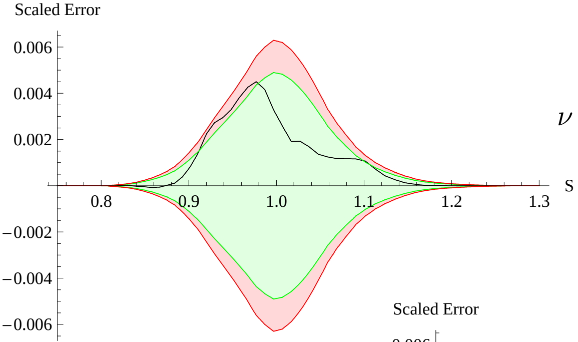
$$D_t = Z_t - \int_0^{+\infty} e^{-s} \, Z_{t-s} \, ds$$

Pricing PDE (degenerate parabolic):

$$a(x, y, t)\partial_{xx} + x\partial_y + \partial_t, \qquad (x, y, t) \in \mathbb{R}^3$$

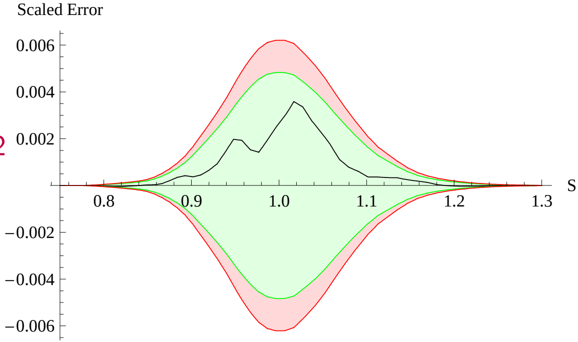
- Complete model - hypoelliptic PDE

Path dependent volatility



$\nu = 20\%\sqrt{2x^2+1},\ D_0 = 0$

$\nu = 20\%\sqrt{2x^2+1},\ D_0 = -0.2$



Path dependent volatility

	$D_0 = 0$		$D_0 = -0.2$	
	MC	Parametrix	MC	Parametrix
$S = 0.8$	0.00065	0.00067	0.00061	0.00069
$S = 0.9$	0.00820	0.00819	0.00904	0.00926
$S = 1$	0.04003	0.03991	0.04302	0.04299
$S = 1.1$	0.09982	0.09980	0.10256	0.10250
$S = 1.2$	0.16806	0.16810	0.16929	0.16927
$S = 1.3$	0.23093	0.23094	0.23129	0.23126

Conclusions

PROs

- **simple explicit formulas** for plain vanilla options $\hookrightarrow$ fast calibration

CONs

- difficult to compute higher order terms

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- **simple explicit formulas** for plain vanilla options $\hookrightarrow$ fast calibration
- **rigorous justification** of the method: asymptotically exact as $t \to 0$ and $|x| \to \infty$

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Conclusions

PROs

- **simple explicit formulas** for plain vanilla options $\hookrightarrow$ fast calibration
- **rigorous justification** of the method: asymptotically exact as $t \to 0$ and $|x| \to \infty$
- expansion using as starting point the Black&Scholes formula

CONs

- difficult to compute higher order terms
- large theoretical error bounds
- works with uniformly parabolic PDEs $\hookrightarrow$ Gaussian type densities

CEV: $\alpha = \frac{3}{4}$, $S \sim \frac{1}{100}$, strike $K = 1$