# Stressing Rating Criteria Allowing for Default Clustering: the CPDO case (*)

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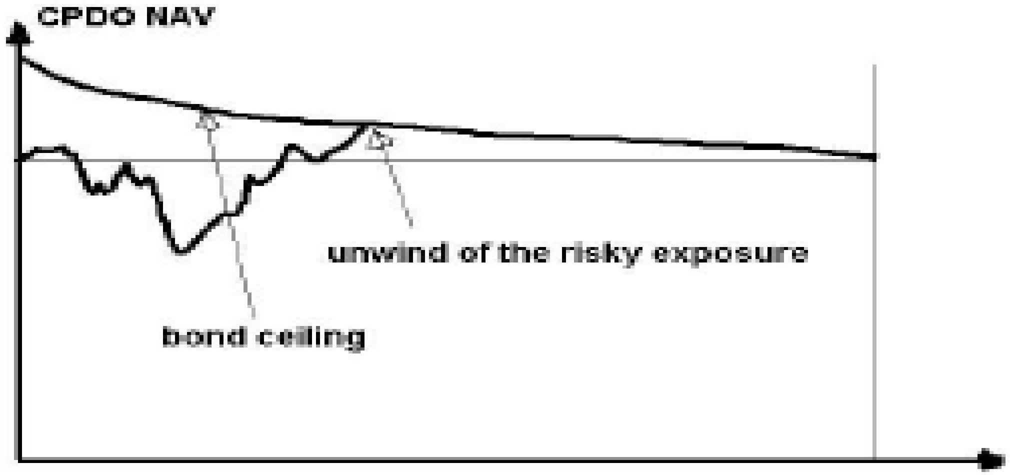
(*) Joint work with Andrea Pallavicini. available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1077762

(**) Credit Derivatives Structuring – Global Markets – BBVA

# Constant Proportion Debt Obligation (CPDO)

- A CPDO is a bond paying a spread over Libor, as much as 2% in some cases, that agencies used to rate AAA.
  - The spread is financed by a strategy that sells unfunded levereged protection on a credit index trying to exploit the mean reverting properties of credit spreads: when the spread widens, and thus the strategy incurs a loss, the CPDO strategy increases the bet.
  - As soon as the NAV of the strategy is sufficient to guarantee the payment of the remaining fees, coupons (Libor plus spread) and principal, the risky exposure is completely unwound.

**CPDO Bond Ceiling and NAV**

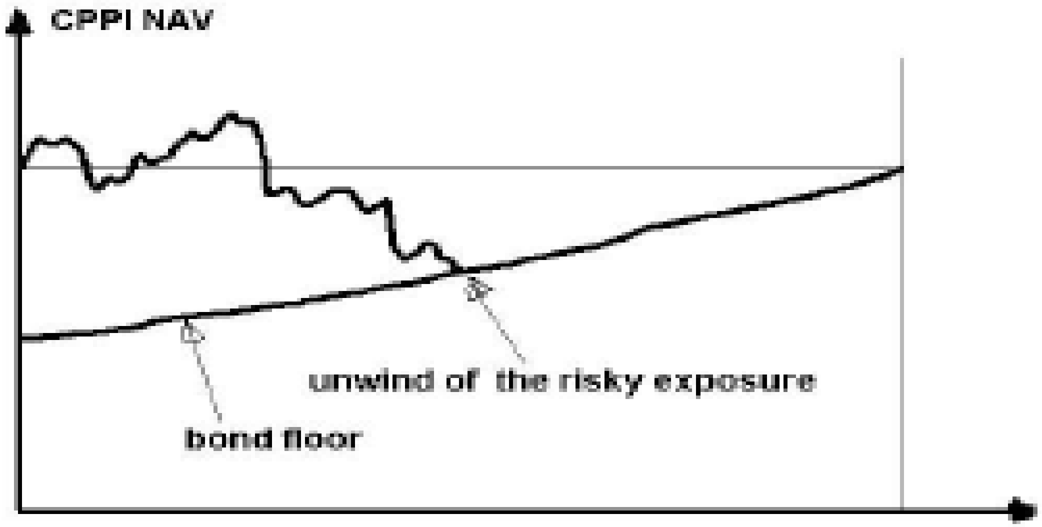



*Source: Torresetti and Pallavicini (2007)*

# Constant Proportion Portfolio Insurance (CPPI)

- CPPI can be seen as a dynamic strategy aiming to preserve the initial capital.
  - The idea is to take leveraged exposure to credit whilst being able at all times to unwind all risky exposure and buy a risk free zero coupon bond.
  - So when spread widens, contrary to a CPDO strategy, the CPPI strategy reduces the leverage.

**CPPI Bond Floor and NAV**



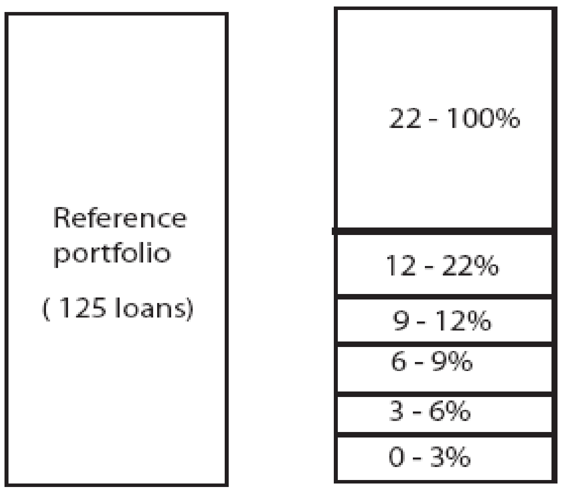

*Source: Torresetti and Pallavicini (2007)*

# CPDO vs CPPI

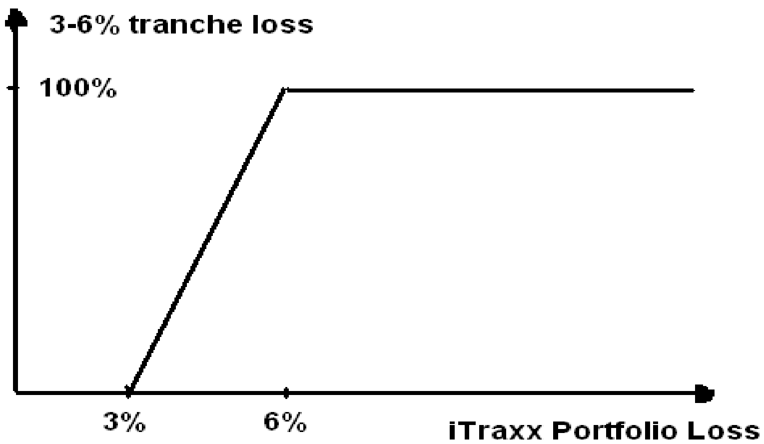
- In a CPDO if the spread widens (the mark to market of the protection sold becomes negative) we increase the leverage.
  - In this way CPDOs try to exploit the mean reverting behaviour that corporate spreads have historically displayed.
- Both are designed for a risk adverse investor
  - CPPI having a capital guarantee attached and CPDO until recently receiving high rating.
- Neverhteless the risks associated to the two strategy are inherently different.
  - the minimum return on capital of a CPPI is 0 % whereas the minimum return of a CPDO can be -100% …
  - … thus the distribution of the returns of the CPPI strategy is right skewed whereas the distribution of the returns of a CPDO strategy is left  skewed.

# Implied Distribution of Credit Portfolio Losses

- Following the spirit of Breeden and Litzenberger (1978), we will extract the implied loss distribution from market prices: more precisely standardized CDO tranches.
  - A credit portfolio loss is sliced into different tranches corresponding to different levels of seniority in the capital structure
    - The oustanding notional of the tranche 0-3% will be reduced as soon as the pool experience losses and will be completely wiped out when the pool loss reaches 3%.
    - The outstanding notional of the tranche 3-6% will be reduced as soon as the pool losses reaches 3% and will be completely wiped out when the pool loss reaches 6%




**3-6% tranche loss versus Portfolio loss**



# Implied Correlation

- Implied correlation is the CDO tranche equivalent of implied volatility for equity options.
  - The market pricing standard for CDO tranches has become the one factor Gaussian copula
    - As for Equity Option the market standard is to quote the implied volatility for different option strike …
    - … for CDO tranches the market standard is to quote different correlations for different tranches

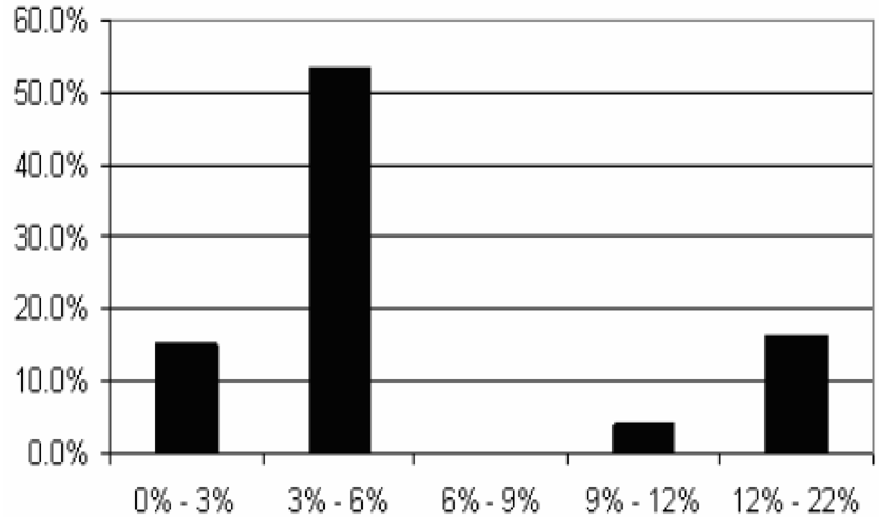
**CDO tranche quotes (left table) and compound correlation (right plot)**

Reference Swap: Itraxx Europe S5

| maturity | ref index |
|---|---|
| 3y | 21 bp |
| 5y | 36 bp |
| 7y | 46 bp |
| 10y | 56 bp |

Tranche Spread: Itraxx Europe S5 10y

| tranche | running | upfront |
|---|---|---|
| 0-3 | 5.00% | 49+.00% |
| 3-6 | 3.60% | 0.00% |
| 6-9 | 0.82% | 0.00% |
| 9-12 | 0.46% | 0.00% |
| 12-22 | 0.31% | 0.00% |

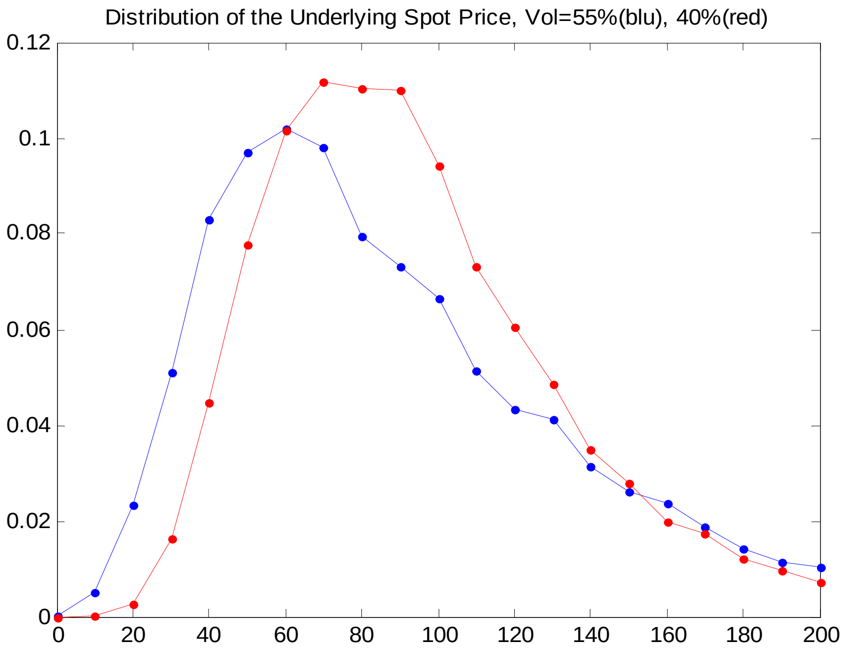



*Source: Torresetti, Brigo and Pallavicini (2006a)*

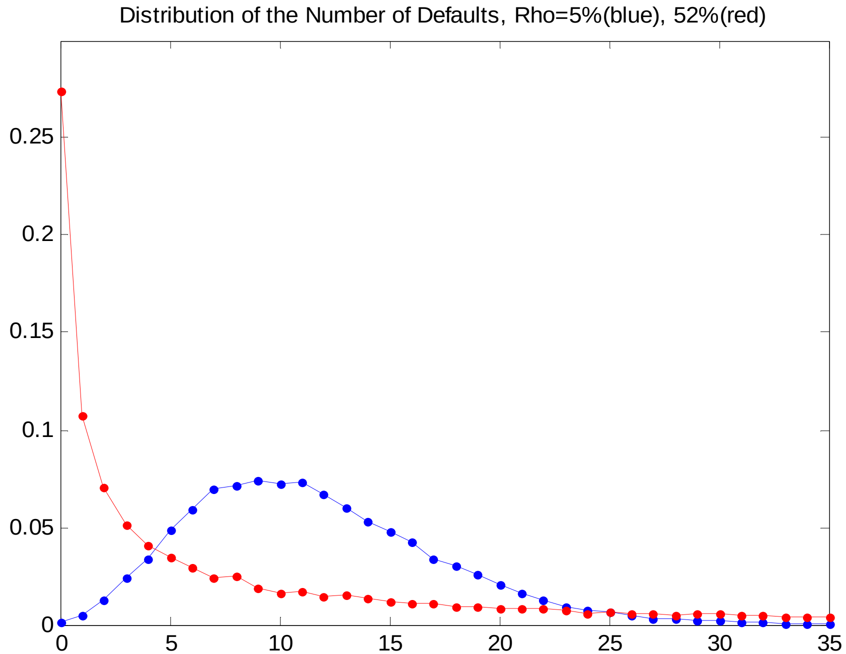
# Implied correlation and shape of the loss distribution

- Contrary to equity options we do not get any intuition regarding the shape of the loss distribution
  - Equity Option: different strike, and correspondingly different implied volatilities, are associated to distributions of the underlying (spot price at option maturity) relatively similar.
  - CDO tranches: different attachment, and correspondingly different implied compound correlation (gaussian copula), are associated to completely different shapes of the underlying (portfolio loss at tranche maturity) distribution.

**Equity Index distribution at option matuirty**




**Distribution of the pool number of default at the CDO tranche maturity**

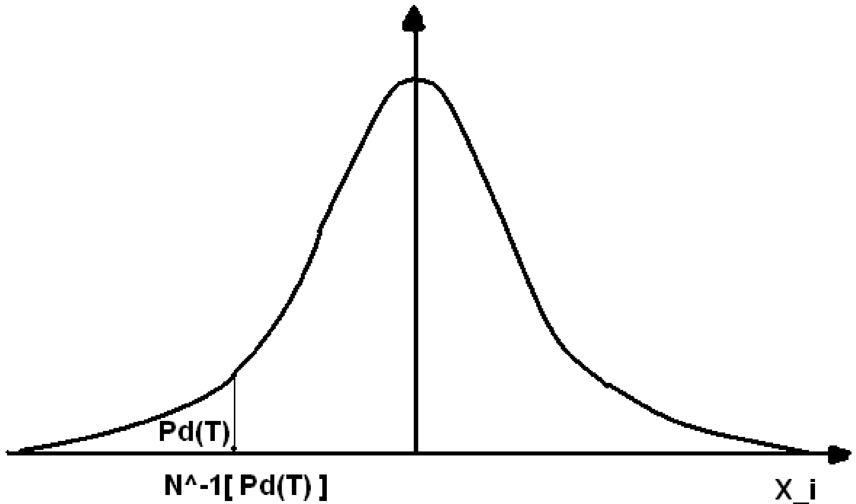


# One Factor Gaussian Copula

- Implied correlation is the CDO tranche equivalent of implied volatility for equity options.
  - consider 125 latent variables correlated through a systemic factor:

$$X_i = Y_i \sqrt{1-\rho} + M\rho \ , \quad i = 1,\ldots,125$$

- where $M$ and $Y_i$ , $i = 1,\ldots,125$ are independent Gaussian random variables.




- conditional on the Gaussian systemic factor $M$ the probability of default is:

$$\text{ProbabilityRiskNeutral}\{\text{Name defaults before } T \mid M\} = N\left(\frac{N^{-1}(PD(T)) - M\sqrt{\rho}}{\sqrt{1-\rho}}\right)$$

# One Factor Gaussian Copula

- Under the homogeneity and large pool assumptions this is also the certain pool default rate at time $T$ given $M$

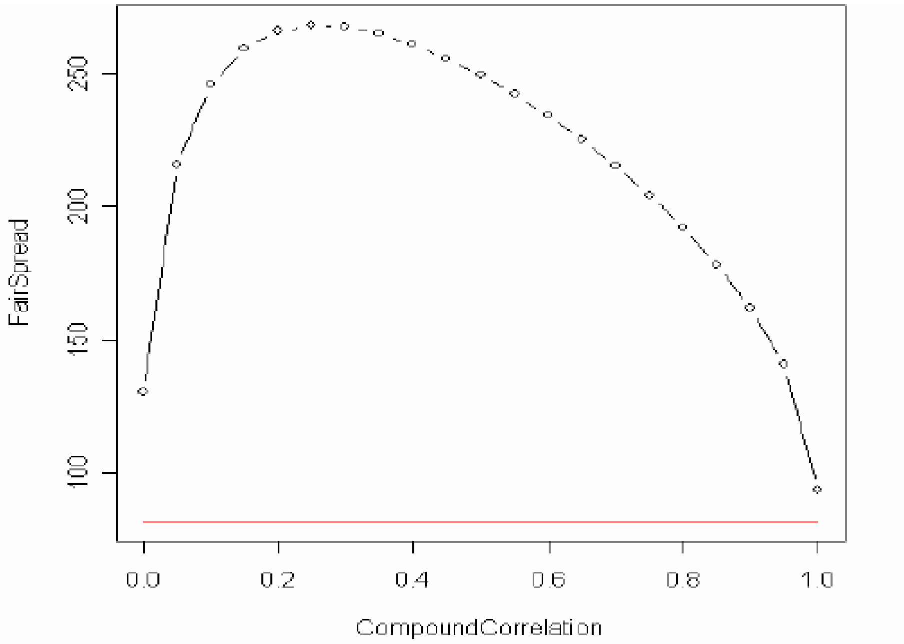
$$\text{One factor Gaussian Copula} \begin{cases} \text{Systemic Scenario} & \text{Scenario Probability} & \text{Conditional default rate} \\ \dots & \dots & \dots \\ \mathrm{M} \in [\mathrm{m}; \mathrm{m} + \mathrm{dm}) & \mathrm{N}(\mathrm{m} + \mathrm{dm}) - \mathrm{N}(\mathrm{m}) & N\left( \dfrac{N^{-1}(Pd(T)) - \mathrm{m}\sqrt{\rho}}{\sqrt{1-\rho}} \right) \\ \dots & \dots & \dots \end{cases}$$

- Integrating across the systemic factor $M$ will give the pool default rate distribution given the latent factor correlation $\rho$

# Implied compound correlation invertibility

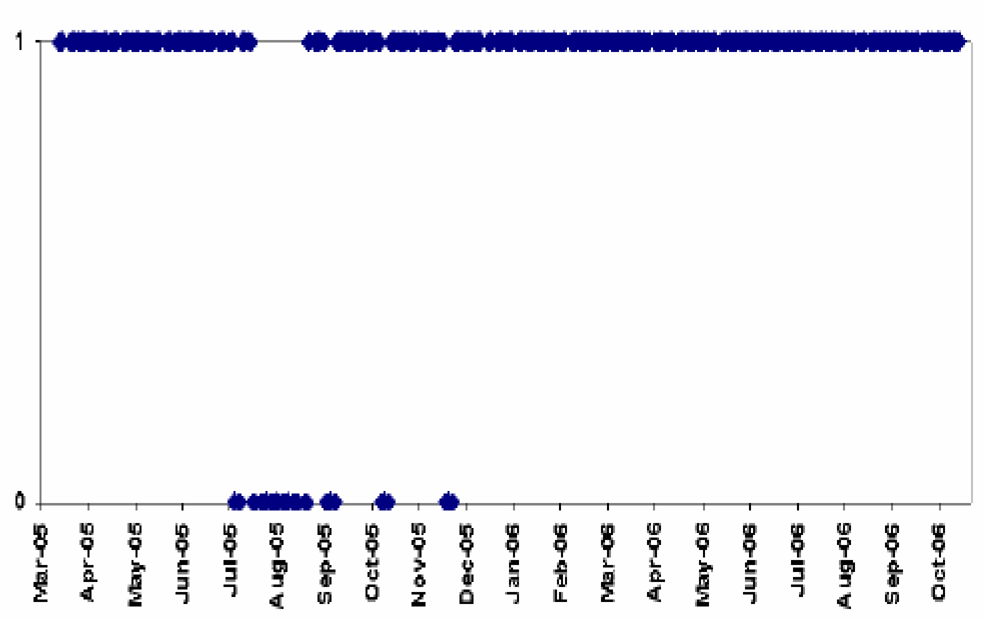
- Not always the compound correlation is invertible
  - The 6-9% 10y tranche on iTraxx and CDX is not invertible under all market conditions

**6-9% tranche spread vs compound correlation**




*Source: Torresetti, Brigo and Pallavicini (2006a)*

**Invertibility Indicator for the iTraxx 10y 6-9% tranche (1 = invertible, 0 = not invertible)**




*Source: Torresetti, Brigo and Pallavicini (2006a)*

# Base Correlation overtakes Compound Correlation

- The expected tranche loss of tranche A-B entering the calculation is computed as a combination of the expected equity tranche losses 0-A and 0-B

The tranched loss can be written as:

$$[1] \qquad L(A,B) = \min\left[B-A, \max\left[L-A,0\right]\right]/(B-A)$$

where L is the portfolio Loss at maturity and A and B are the attachment and detachment points.

With a little manipulation we can write:

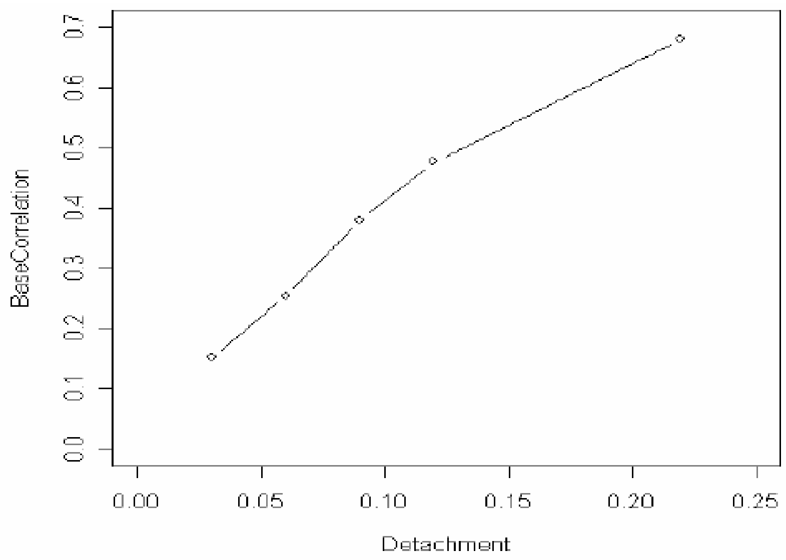
$$[2] \qquad L(A,B) = (\max\left[L-A,0\right] - \max\left[L-B,0\right])/(B-A)$$
$$= (-\min\left[0,A-L\right] + \min\left[0,B-L\right])/(B-A)$$
$$= (-\min\left[L,A\right] + L + \min\left[L,B\right] - L)/(B-A)$$
$$= (B \cdot L(0,B) - A \cdot L(0,A))/(B-A)$$

- If we were to price a bespoke 1.5-4.5% tranche we would not know which compound correlation to use.
  - Interpolate 16% (0-3% correlation) and 52% (3-6% correlation)?
  - What if it was a bespoke 1.5-12% tranche?

# Base Correlation

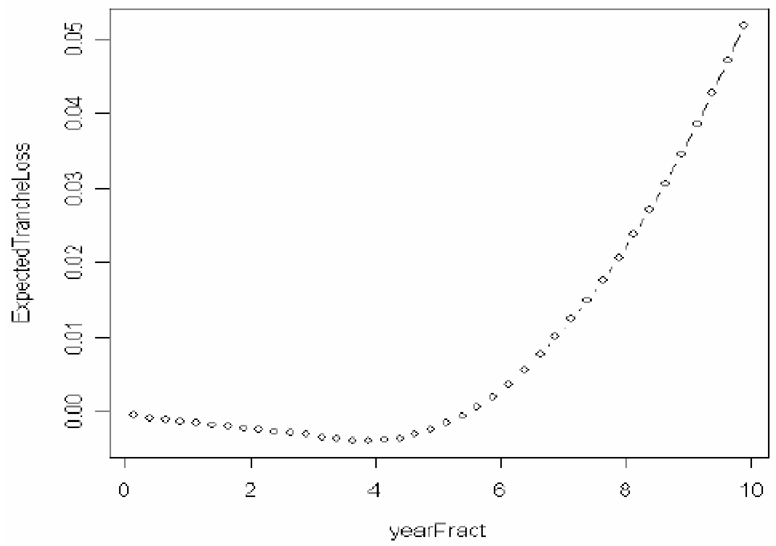
- Given that to calibrate the 0-3% tranche we computed the expected value of L(0,3%)
  - Then we seek the compound correlation for which the expected value of L(0,6%) is such that the expected value of L(3%,6%) calibrates the 3-6% tranche.
- The base correlation mapping is more smooth
  - we can price bespoke tranches via interpolation
- Still it is inconsistent as it implies that the expected tranche loss could be negative

**Base Correlation**




*Source: Torresetti, Brigo and Pallavicini (2006a)*

**9-12% expected tranche loss E[ L(9%, 12%) ]**




*Source: Torresetti, Brigo and Pallavicini (2006a)*

# Implied Copula versus Gaussian Copula

- Replace the parametric formula, dependent upon the correlation, for the default rate conditional on the realization of the systemic factor …

$$\text{ProbabilityRiskNeutral}\{\text{Name defaults before } T \mid M\} = N\left(\frac{N^{-1}(PD(T)) - M\sqrt{\rho}}{\sqrt{1-\rho}}\right)$$

- With the more natural intensity based formula

$$\text{ProbabilityRiskNeutral}\left\{\text{Name defaults before } T \mid M = m^S\right\} = 1 - e^{-\lambda^S T}$$

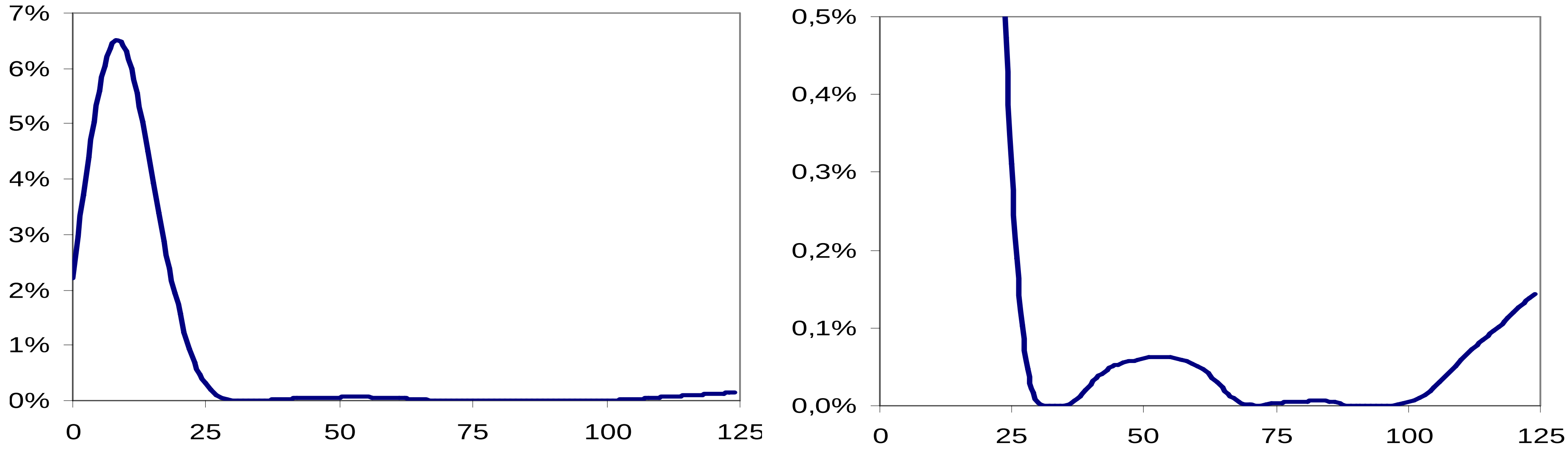
- where the systemic factor realization is interpreted directly as pool default rate

$$\text{Implied Copula}\begin{cases} \text{Systemic Scenario} & \text{Scenario Probability} & \text{Conditional default rate} \\ \mathrm{M} = \mathrm{m}^0 & p^0 & 1 - e^{-\lambda^0 T} \\ \mathrm{M} = \mathrm{m}^1 & p^1 & 1 - e^{-\lambda^1 T} \\ \ldots & \ldots & \ldots \\ \mathrm{M} = \mathrm{m}^{124} & p^{124} & 1 - e^{-\lambda^{124} T} \end{cases}$$

# Implied Copula versus Gaussian Copula

- In the One Factor Gaussian Copula case this distribution has little flexibility, in that one can play only with the single copula parameter $\rho$ , scenario probabilities being fixed by the Gaussian assumption.
  - Each different CDO tranche is calibrated via a different correlation parameter (different underlying portfolio loss distribution)
- In the Implied Copula approach instead we calibrate the scenario probabilities $p^i,\ i = 0,1,\ldots,124$ so as to calibrate all CDO tranches at the same time with the same distribution

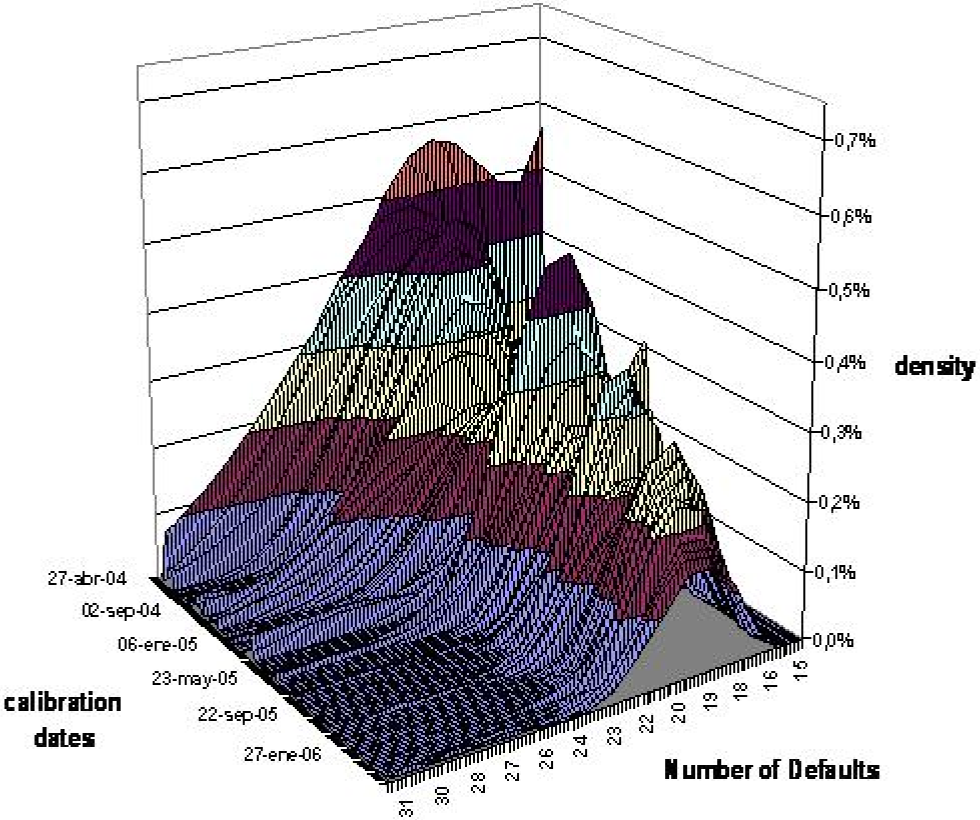
**Implied 10 year Default Rate Distribution for the iTraxx pool on 14-jun-2005**

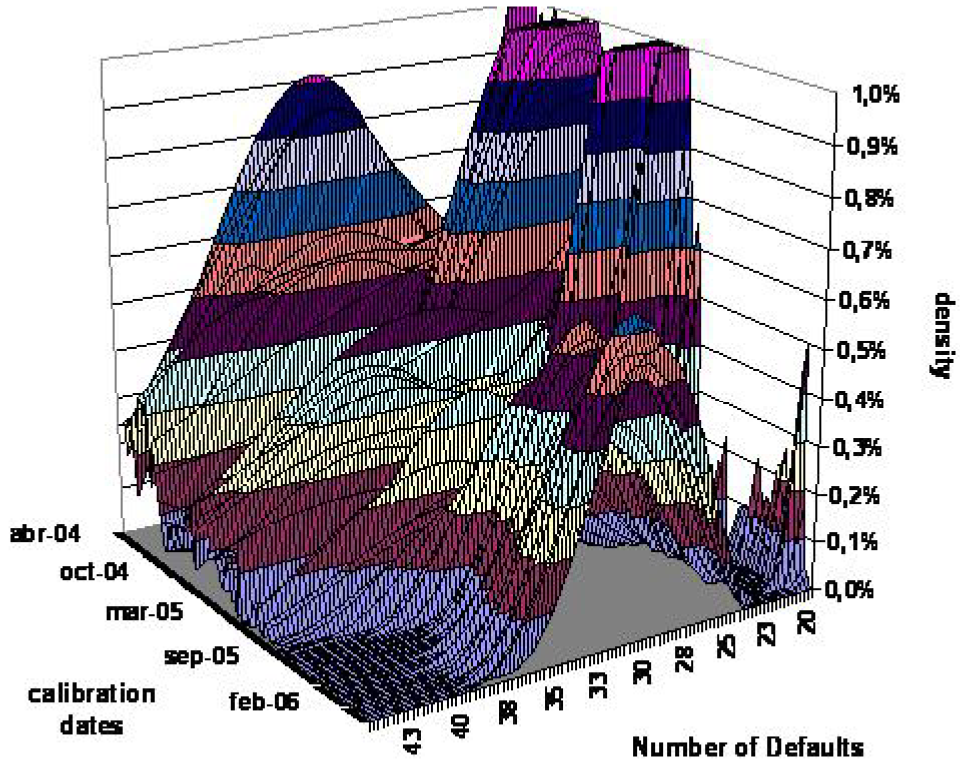


# Right tail bumps in the Implied Default Rate Distribution

- The right tail bump is a persistent feature of default rate distributions implied from CDO tranches across maturities (10 year and 5 year) and Regions (iTraxx and CDX).

**Implied 5 year Default Rate Distribution for the iTraxx (left) and CDX (right) pool**



# Dynamical Default Rate Models

- The Bump Feature was embedded into a dynamical model for the number of defaults of a pool of names that aims to reprice all tranches across all strike and across also all maturities.
  - Brigo, Pallavicini and Torresetti (Risk, May 2007)
- A dynamical model for the number of defaults of a pool of names, modelling the number of defaults as a linear combination of independent Poisson processes with different intensities or, in other terms, as a Generalized Poisson Loss (GPL) process.

$$Z_t = N_1(t) + 2N_2(t) + 3N_3(t) + \ldots + nN_n(t).$$

$$C_t := \min(Z_t, M) = Z_t 1_{\{Z_t < M\}} + M 1_{\{Z_t \geq M\}}$$

# Generalized Poisson Loss (GPL) process

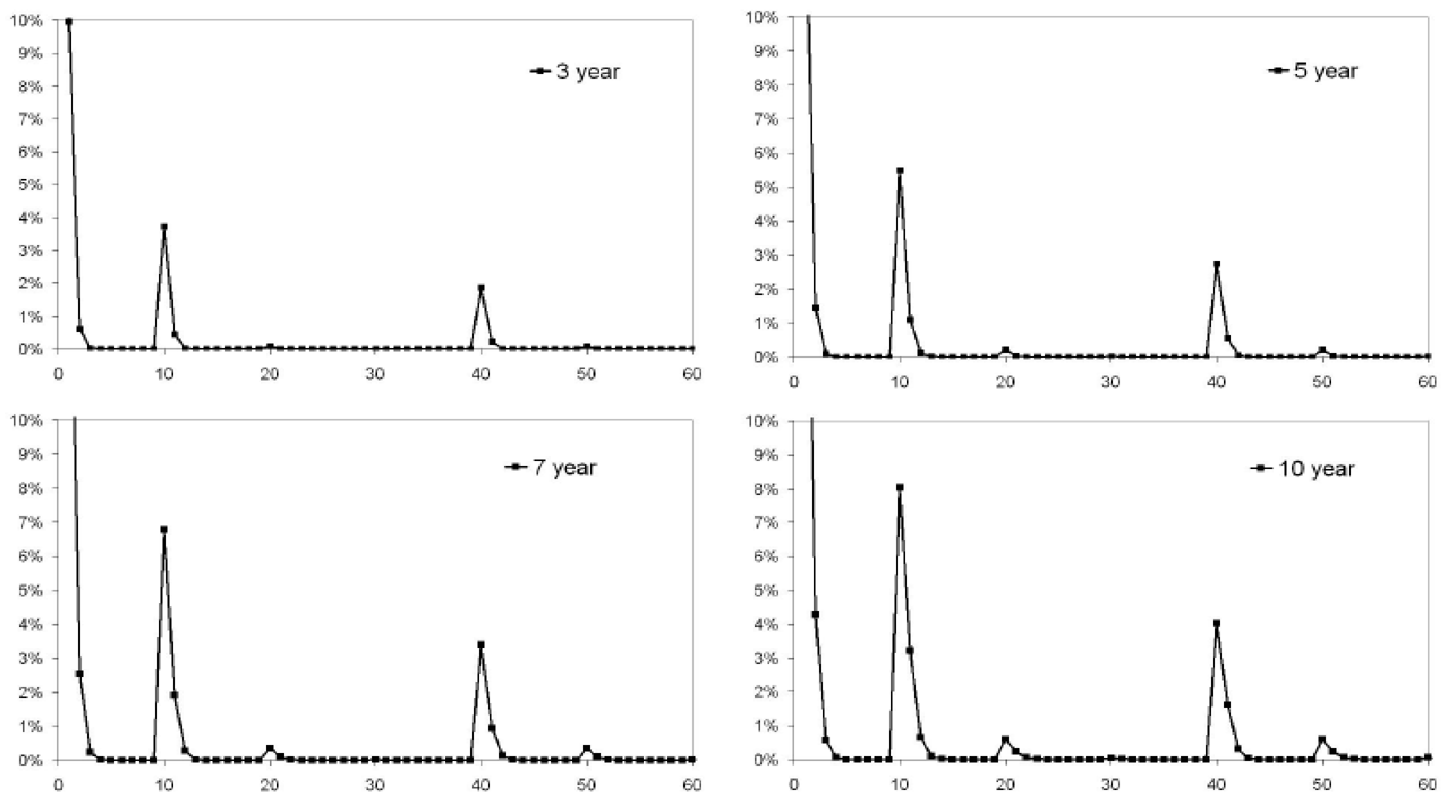
- Take a pool of 4 names (*n*=4)
  - with constant intensities $\lambda_i$ for the poisson processes $N_i$ and $\lambda_2 = \lambda_4 = 0$
- The generator of the transition matrix for the default count process $C_t$ is:

$$A = \begin{bmatrix} -(\lambda_1 + \lambda_3) & \lambda_1 & 0 & \lambda_3 & 0 \\ 0 & -(\lambda_1 + \lambda_3) & \lambda_1 & 0 & \lambda_3 \\ 0 & 0 & -(\lambda_1 + \lambda_3) & \lambda_1 & \lambda_3 \\ 0 & 0 & 0 & -(\lambda_1 + \lambda_3) & \lambda_1 + \lambda_3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- The first line of $\exp(T \cdot A)$ is going to be the probability distribution of observing 0 to 4 defaults by time *T* given that the initial default count is 0.

# Generalized Poisson Loss (GPL) process

- Take also the following example (*n*=125)
  - all lambdas are set to zero except for $\lambda_1 = 0.04, \quad \lambda_{10} = 0.015, \quad \lambda_{40} = 0.0075$
- The default count distribution at 3, 5, 7 and 10 year is going to be:



# Generalized Poisson Cluster Loss (GPCL) process

- In the GPL all poisson processes can keep on jumping even after the pool default count is saturated.
  - In the GPCL at each point in time we check for each cluster of defaults if, given the current default count, the remaining pool size allows for the default of that cluster

$$C_t = \sum_{j=1}^{M} j\, Z_j(t) \qquad dZ_j(t) \approx \begin{cases} \text{Poisson}\left(\binom{M - C_{t^-}}{j} \lambda_j(t)dt\right) & \text{if} \quad \mathrm{j} \leq M - C_{t^-} \\ 0 & \text{if} \quad \mathrm{j} > M - C_{t^-} \end{cases}$$

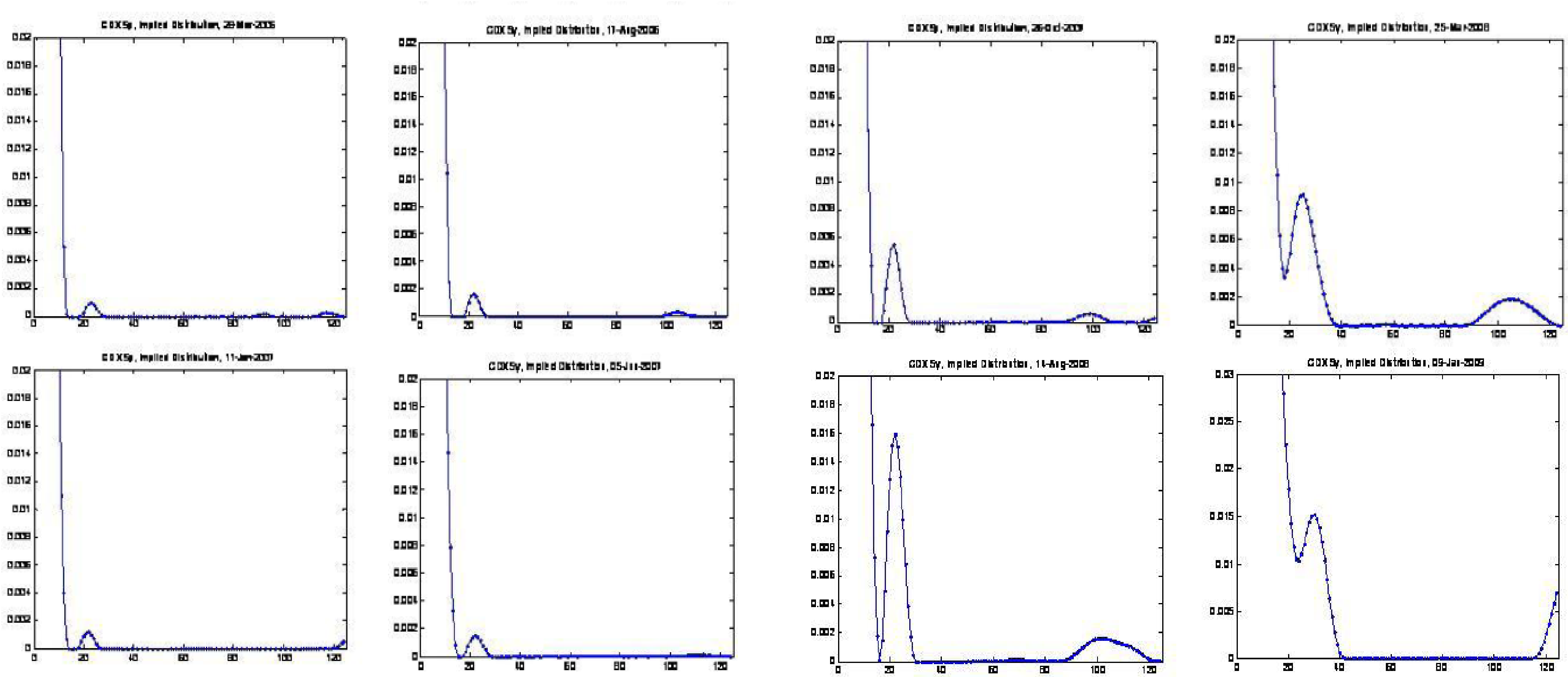
# Generalized Poisson Cluster Loss (GPL) process

- Take a pool of 4 names (*n*=4)
  - with constant intensities $\lambda_i$ for the poisson processes $N_i$ and $\lambda_2 = \lambda_4 = 0$
- The generator of the transition matrix for the default count process $C_t$ is:

$$A = \begin{bmatrix} -a_1 & \lambda_1 \binom{4}{1} & 0 & \lambda_3 \binom{4}{3} & 0 \\ 0 & -a_2 & \lambda_1 \binom{3}{1} & 0 & \lambda_3 \binom{3}{3} \\ 0 & 0 & -a_3 & \lambda_1 \binom{2}{1} & 0 \\ 0 & 0 & 0 & -a_4 & \lambda_1 \binom{1}{1} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

# Bumps in the risk-neutral measure

- How reasonable is it to assume a loss distribution in the **risk neutral** measure with bumps?
  - If reasonable, how reasonable is it to assume a constant jump amplitude in the calibration procedure through time



# Bumps in the objective measure

- How reasonable is it to assume a loss distribution in the **objective** measure with bumps?
  - Probability bumps in the right tail associated to default clusters could be just a premium required by investors for holding the skewed risk of senior tranches
- Supporting Evidence
  - Clusters of defaults are rare events even though historically we have seen a sequence of defaulted names belonging to the same sectors in a relatively short amount of time
    - savings and loan crisis (1990), airlines (2001-2002) and auto part makers and financials recently.
  - Longstaff and Rajan (2007) ran a Principal Component Analysis on the CDS spread changes of the constituents of the CDX index.
    - First component: is a systematic shock affecting all CDS (poisson process governing the jump to default of the entire pool)
    - Remaining signifcant components: shocks affecting only one sector at a time (poisson process governing the jump to default of a number of entities equal to the average sector size)
    - The remaining variance is idiosyncratic risk: (poisson process governing the jump to default of one entity at a time)

# Anatomy of a CPDO

- A CPDO is a note bond paying the investor an interest until the earliest of:
  - Maturity. Usually 10 years to give enough time to the structure to profit via a dynamic strategy from the mean reverting properties of the underlying credit derivatives index spread.
  - Cash-in. The time when the structure NAV touches the Bond Ceiling.
  - Cash-out. In case the structure NAV reaches 10% the leveraged credit exposure is unwound and the proceed if any are given back to the investor.

# Anatomy of a CPDO

- Dynamic leverage rule (continuously rolled in the on-the-run credit index series)

$$\beta_t = \begin{cases} 7.5 & \text{if} \quad \max\limits_{s\in[0,t]} S_s < 0.40\% \\ 10 & \text{if} \quad 0.40\% \leq \max\limits_{s\in[0,t]} S_s < 0.50\% \\ 12.5 & \text{if} \quad 0.50\% \leq \max\limits_{s\in[0,t]} S_s < 0.60\% \\ 15 & \text{if} \quad 0.60\% \leq \max\limits_{s\in[0,t]} S_s \end{cases}$$

  - Leverage rule overridden in case the NAV of the CPDO below 40%, then leverage is reduced until it reaches 2-3 when the NAV is around 10%. The strategy unwinds completely the risky exposure as soon as the NAV falls below 10%.

- Spread over Libor received by the CPDO note holder

$$I_t = \begin{cases} 1.25\% & \text{if} \quad \beta_t = 7.5 \\ 1.5\% & \text{if} \quad \beta_t = 10 \\ 1.75\% & \text{if} \quad \beta_t = 12.5 \\ 2\% & \text{if} \quad \beta_t = 15 \end{cases}$$

# Anatomy of a CPDO

- Structure Fees: Management, Gap and Upfront Fees.

$$X_t = \left(X_t^{\mathrm{MF}} + \beta_t X_t^{\mathrm{GP}}\right) + 1_{\{t=0\}} X^{\mathrm{UF}}$$

- Bond Ceiling
  - In case the structure NAV reaches the bond ceiling $B_t$ the leveraged credit exposure is unwound and the proceeds are invested in a basket of risk free bonds that will guarantee the payment of the remaining fees, interest plus spread and the principal at maturity.

$$B_t = 1 + \int_t^{T_I^{40}} (X_s + I_t) D(t,s) ds$$

# Anatomy of a CPDO

- At inception these are the events following the investors' subscription of the CPDO notes
  - The arranger takes the upfront fee $X^{\mathrm{UF}}$
  - The structure puts the remainder in a short term deposit
  - The vehicles sells protection on a credit derivatives index for a notional initially of 7.5 times the notes notional
- On any subsequent date the structure will:
  - Receive libor on the cash invested.
  - Pay libor plus spread on the notes notional to the investor.
  - Pay the Loss Given Default (LGD) times the leverage for any name defaulted in the underlying credit index to the protection buyer.
  - Receive the protection premium times the leverege from the protection seller.
  - Pay the fees (management and gap-risk) to the arranger.

$$dV_t = V_t r_t dt - (F_t + I_t)dt - \beta_t dL_t + \left( S_{T_R^i} \beta_{T_R^i} + \int_{T_R^i}^{t} S_s d\beta_s \right) dt - X_t dt + dNPV_t^{T_R^i}$$

# Anatomy of a CPDO

We also consider the recovery rate ($R$) to be constant in time and equal to 35%, so that the number of defaults $C_t$ and the pool loss $L_t$ processes are linked by the usual relationship $L_t = (1 - R)C_t$.

Let's call $T_R^i$ the last roll date and $T_I^j$ the last interest payment date of the underlying credit index[6].

The net present value $NPV_t^{T_R^i}$ of the protection sold since the last roll-date $T_R^i$ is:

$$NPV_t^{T_R^i} = \left( S_{T_R^i} \beta_{T_R^i} + \int_{T_R^i}^{t} S_s d\beta_s \right) DV01_t + \beta_t DFLT_t$$

# CPDO Rating Criteria

- To simulate the NAV of a CPDO strategy we need to simulate in the objective measure
  - Interest Rates (**A**)
    - assumed for simplicity deterministic
  - Credit Index Spread (**B**)
    - Modeled as an exponential vasicek with parameters estimated historically
  - Index pool losses (**C**)
    - Modeled with a multifactor gaussian copula or alternatively with a GPCL: in both cases the expected 6 month pool loss is given by the assumed constant rating composition

$$dV_t = \underbrace{V_t r_t dt - (F_t}_{A} + I_t)dt - \underbrace{\beta_t dL_t}_{C} + \underbrace{\left( S_{T_R^i}\beta_{T_R^i} + \int_{T_R^i}^{t} S_s d\beta_s \right)}_{B} dt - X_t dt + dNPV_t^{T_R^i}$$

- Along a simulation path the CPDO strategy will incur in a default when
  - It is not able to deliver the promised coupons and the principal at maturity …
  - … or equivalently if the bond ceiling was reached before maturity.

# CPDO Rating Criteria

- We will see how the rating of CPDO result being extremely sensitive to the underlying set of simulation parameters.
  - Surprisingly it seems that not a stressed set of assumptions but rather an average set of assumptions was used when assigning a rating to a CPDO
- Following Linden et al. (2007) we will compare a Base Case, that can be considered close enough to the criteria used by agencies to assign a rating to CPDOs, to a Stressed Case across several dimensions:
  - Credit spread process parametrization
  - Roll Down Benefit
  - Pool loss simulation
    - In particular we will introduce the possibility of a loss distribution admitting cluster of defaults over any short time interval as in the GPL and GPCL.
- Finally we will see how using GPCL to simulate the loss, results in more penalizing provisions for the gap risk embedded in CPDO structures.

# CPDO Rating Criteria: Credit Index Spread

- Following Fitch's approach described in Linden et al. (2007), we model the on-the-run index spread as an exponential Vasicek process under the objective measure.

$$dS_t = \alpha S_t(\theta - \ln S_t)dt + \sigma S_t dW_t$$

- The index position is invested at any time in the on-the-run series
  - We will model only the 5 year maturity and will adjust the simulated spread in between any two roll dates to take into account the average slope of investment grade curves
- At the time CPDO were rated the time series of credit index spread was limited to 3 years (from 2004 to 2007). Also this time window was limited to a bull market for credit.
  - The limited history available for the iTraxx and CDX indeces can be overcome back-filling the sample with a proxy.
  - This proxy was built from cash bond indices to match the duration, rating and geographical composition of the Globoxx index (50% iTraxx and 50% CDX).

# CPDO Rating Criteria: Credit Index Spread

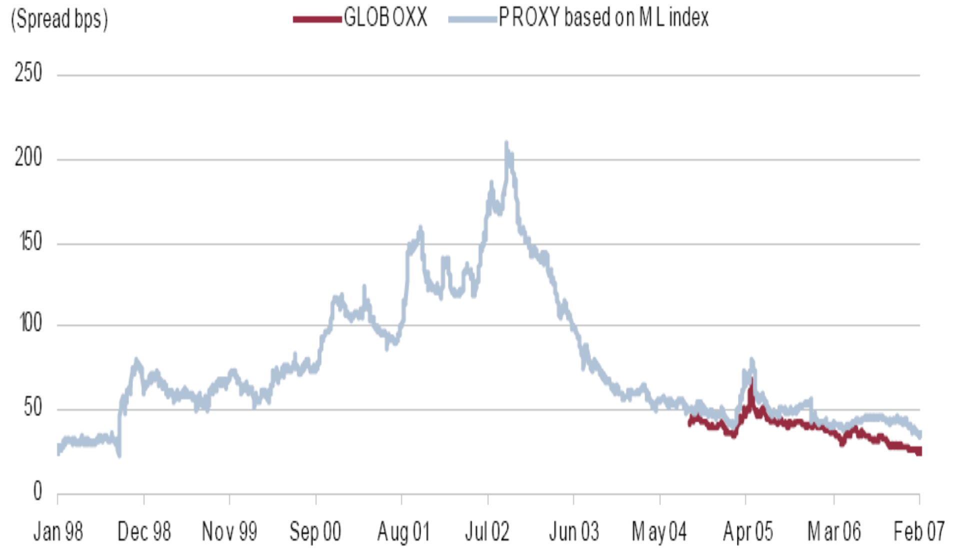
- The parameters estimated on the backfilled indices, and their standard deviations, reported in Linden et al. (2007), are:

**Exponential Vasicek Parameters estimated on the proxied Credit Index**

| | Estimate | 1/4 StdDev |
|---|---|---|
| $\alpha$ | 0.556 | 0.056 |
| $\theta$ | 0.80 % | 0.25 % |
| $\sigma$ | 40.7 % | 1.3 % |

*Source: Linden et al. (2007)*

**Globoxx vs Proxied Index spread**



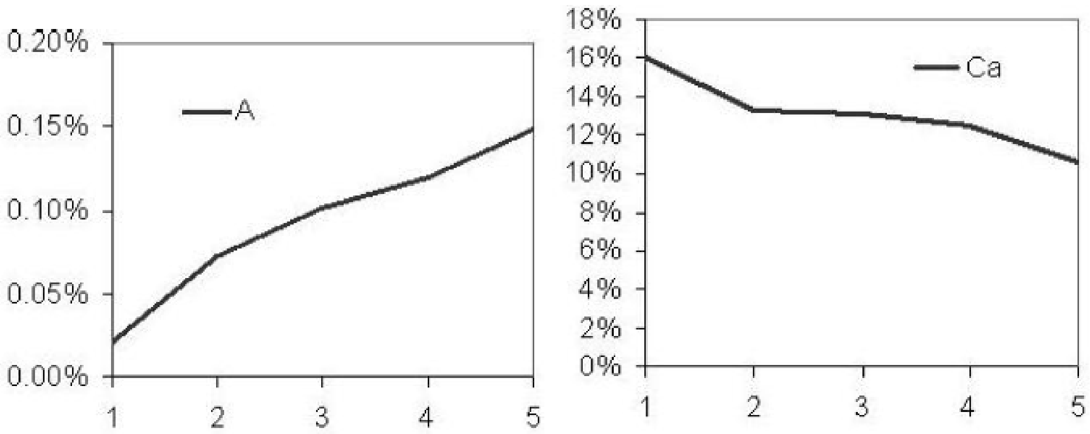

*Source: Linden et al. (2007)*

# CPDO Rating Criteria: Credit Index Spread

- Base Criteria
  - consists in simulating the spread using the mean estimate of the process parameter
- Stressed Criteria
  - consists in taking the mean estimate and stress it by a quarter of the estimate standard deviation so that the CPDO strategy is penalized, i.e. that the bond ceiling is reached less frequently:
    - decrease the mean reversion level (theta),
      - so that on average the dynamic strategy accrues less
    - decrease the mean reversion pull (alpha),
      - So that credit spreads could stay wide for long increasing the probability of cashing-out
    - increase the instantaneous volatility (sigma).

# CPDO Rating Criteria: Roll Down Benefit

- The Roll Down Benefit (RDB) is the difference between the 5y 3m spread of the newly rolled on the run index and the 4y 9m spread of the on the run index CDS spread.
  - Agencies' criteria for CPDO have assumed to some extent that on each roll when closing the short protection position on the old series the strategy would make a gain.
  - The underlying assumption being that investment grade curves are on average positively sloped.
- The benefit however is not certain as the difference in spread can be explained by the different rating composition of the two indices

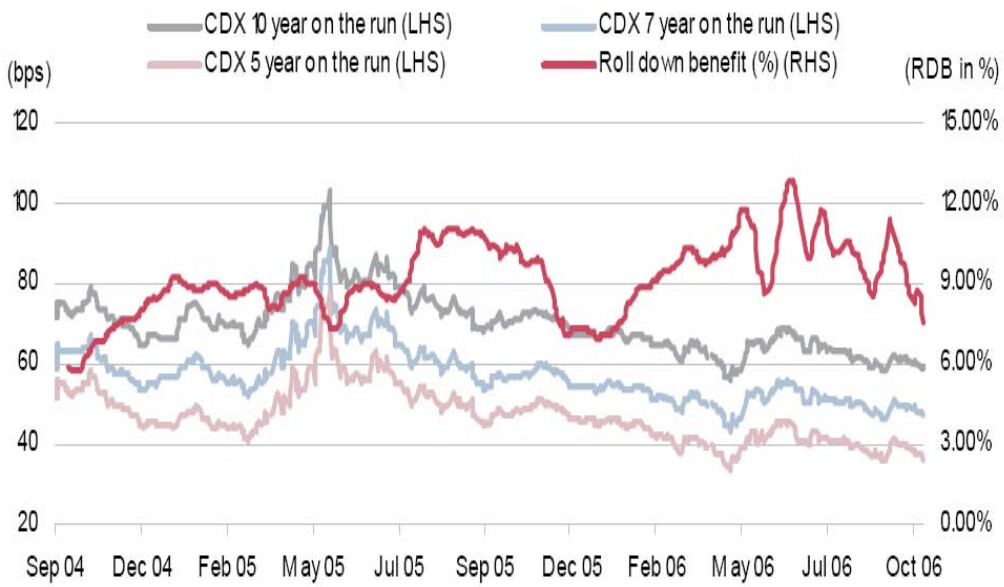



**Default intensity for A rated (left chart) and Ca rated (right chart) corporate credit references.**

# CPDO Rating Criteria: Roll Down Benefit

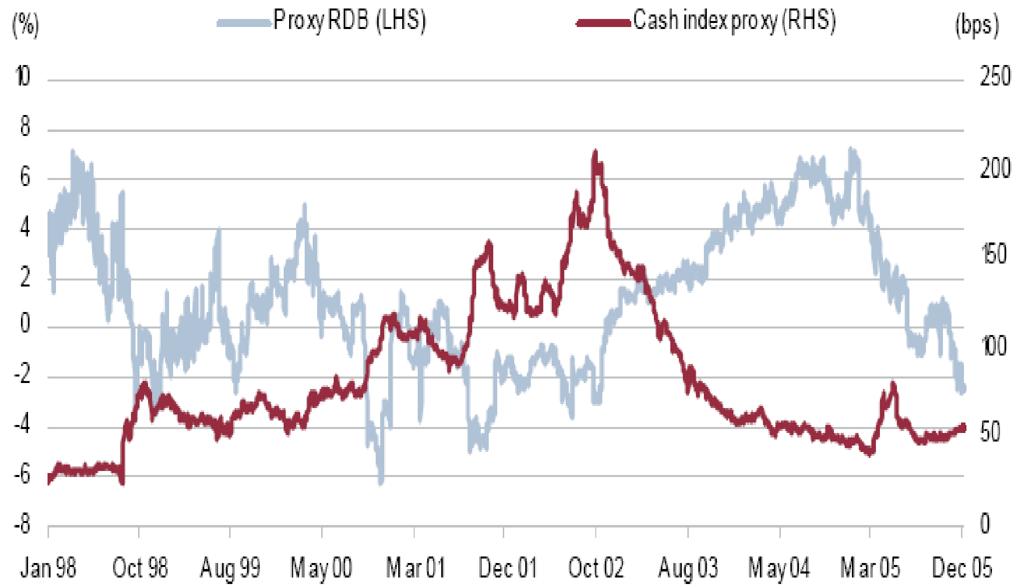
- The term structure of IG names is positively sloped only on average.

**RDB for the CDX index (same pool, 5y3m – 4y9m)**




*Source: Linden et al. (2007)*

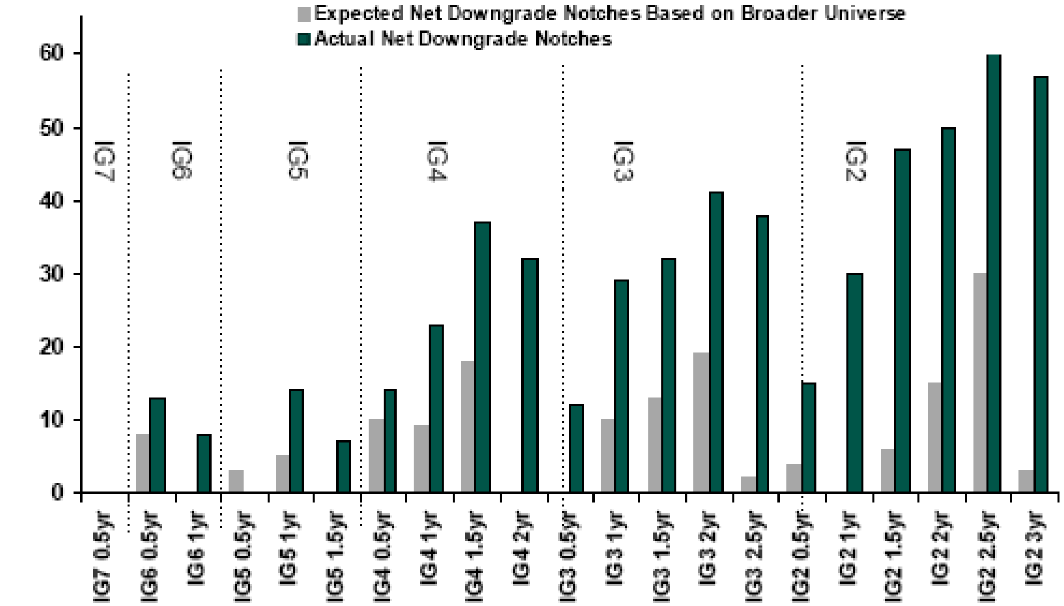
**RDB for the proxied Globoxx index (same pool, 5y3m – 4y9m)**




*Source: Linden et al. (2007)*

# CPDO Rating Criteria: Roll Down Benefit

- iTraxx and CDX positive slope might be influenced by adverse selection bias in the indices.
  - Names that could be downgraded sooner rather than later, turn out to be the most actively traded (liquid) in the CDS market:
    - by lenders that could seek protection in front of their exposure
    - by synthetic CDO originator trying to maximize the tranche spread given a tranche rating constraint




Source: Moodys, Lehman Brothers. Analysis period March 2004 through March 2007.
"The expected rating actions were computed by subjecting rating distribution of the CDX portfolios to Moody's rating transition matrix over the same period

**total downgrade notches minus the total upgrade notches across different series of the CDX indices: actual versus expected from transition matrices.**

*Source: Tejwani et al. (2007)*

# CPDO Rating Criteria: Roll Down Benefit

- Base Criteria
  - 3% roll down benefit.
    - Simulate the constant maturity 5y credit spread between any two roll date.
    - Then increase the simulated spread at inception by 1.5% and decrease the simulated spread after 6m (next roll date) by 1.5% and interpolate to obtain the mark-up (from inception to 3m after inception) or mark-down (from 3m after inception to the next roll) on all other dates.
- Stressed Criteria
  - Assume no roll down benefit.
    - In fact we will model only the CDS spread for the 5 year maturity
    - Without admitting the possibility of a curve inversion we would be recognizing a benefit to the strategy without any downside risk counterpart (i.e. without the risk of a loss when rolling the position).

# CPDO Rating Criteria: Pool Losses – Average Loss.

- On roll dates the credit quality of the indices is refreshed. The rating distribution of recent indices is the following:

**Rating Composition at inception of iTraxx and CDX index series**

| | iTraxx | | | | CDX | | | |
|---|---|---|---|---|---|---|---|---|
| | S5 | S6 | S7 | S8 | S6 | S7 | S8 | S9 |
| AAA | 00.8% | 00.0% | 00.0% | 00.0% | 03.3% | 03.3% | 03.2% | 03.2% |
| AA | 09.3% | 13.1% | 11.7% | 16.9% | 02.5% | 02.5% | 02.4% | 02.4% |
| A | 40.7% | 36.9% | 40.0% | 36.3% | 34.2% | 35.8% | 36.0% | 38.4% |
| BBB | 49.2% | 50.0% | 48.3% | 46.8% | 60.0% | 58.3% | 58.4% | 56.0% |

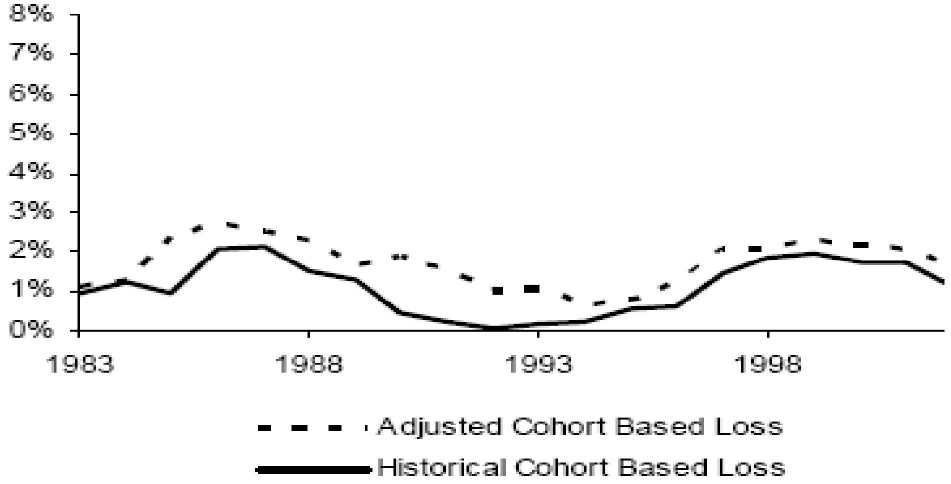
*Source: Torresetti and Pallavicini (2007)*

- We can compute the 6 month default rate on each roll date assuming a constant rating composition at inception and a constant default intensity over the 1st year …
    - This would yield a 6m default rate for the Globoxx of 0.11%.
    - This number could be 60% above the number computed via the rating transition matrix generator estimated on continous rating transition data
    - Desuming default rate from the constituents rating at inception does not take into account the adverse selection that appears to have affected the CDX from 2004 to 2007 as reported in Tejwani et al. (2007)

# CPDO Rating Criteria: Pool Losses – Average Loss.

- The high leverage of the CPDO is also vulnerable to non homogeneity of default rates.
    - evidence that transition matrices are not time homogeneous: there is evidence of rating momentum, see Altman and Kao (1992) and Lucas and Lonski (1992), and evidence that rating transitions differ depending on the credit cycle, see Nickell et al. (2000) and Bangia et al. (2002).

**5-years Rolling Buy-and-Hold Loss for a Portfolio with the same Rating Distribution as the CDX.NA.IG S8.**




*Source: Moody's default data, Lehman Brothers*
*40% recovery used for all calculations*

*Source: Tejwani et al. (2007)*

# CPDO Rating Criteria: Pool Losses – Distribution

- The rating agencies simulation engines for loss-dependent products are based on a multi-factor Gaussian copula (see McGinty and Ahluwalia (2004)).
  - see the S&P's CDO Evaluator 7 and the Fitch's Vector Model.

**Correlation assumptions of the S&P's CDO Evaluator. For example, if two entities are in the same sector but in di erent regions the correlation will be higher (15% rather than 0%) only if the sector is global (technology) rather than local (regulated utilities) or regional**

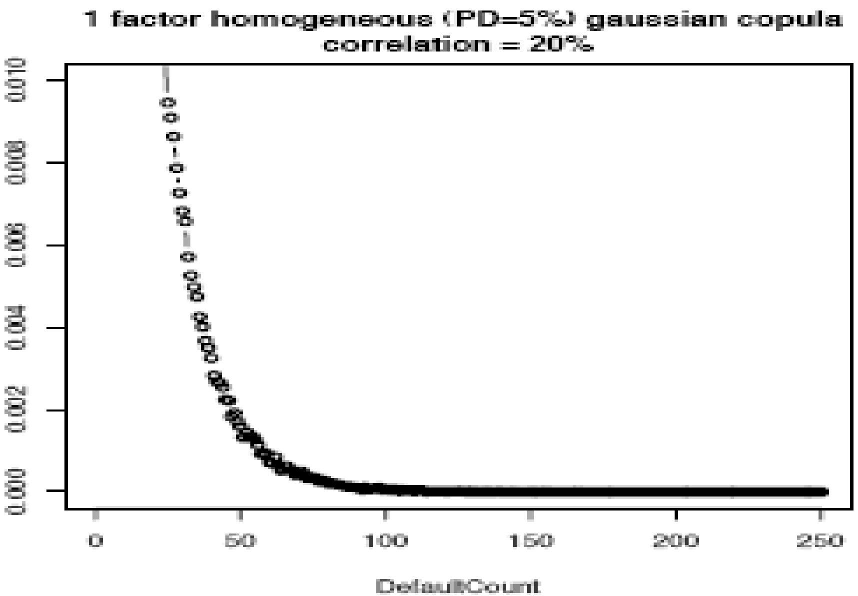
| | Between Sectors | Within Sectors | Sector Type |
|---|---|---|---|
| Within Countries | 0.05 | 0.15<br>0.15<br>0.15 | local<br>regional<br>global |
| Within Regions | 0.05 | 0.05<br>0.15<br>0.15 | local<br>regional<br>global |
| Between Regions | 0.00 | 0.00<br>0.00<br>0.15 | local<br>regional<br>global |

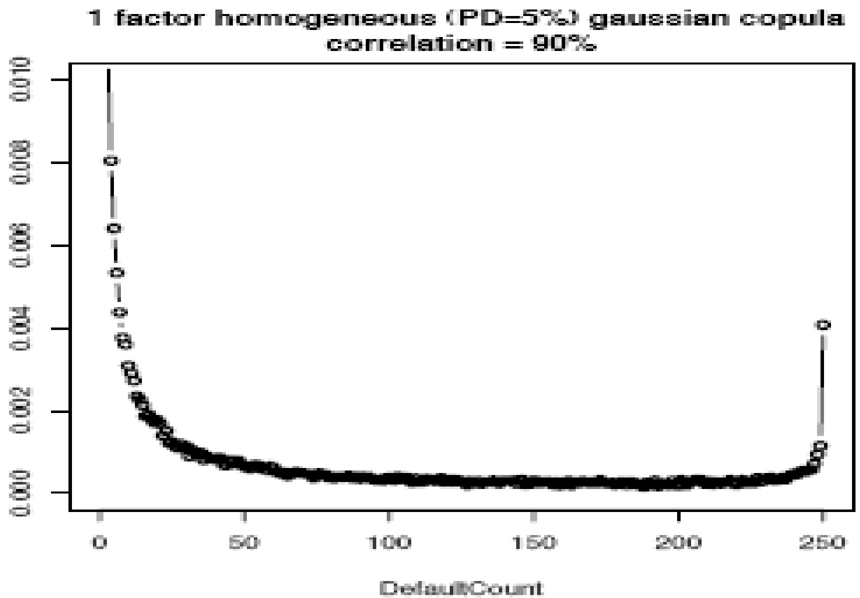
*Source:* *www.standardandpoors.com*

# CPDO Rating Criteria: Pool Losses – Distribution

- Given this correlation assumption it will be quite difficult to simulate loss paths where cluster of defaults occur over a short time horizon.
- Conversely right-tail bumps arise naturally in many statical and dynamical loss models.
  - Among others, multi-modal loss distributions are predicted in Albanese et al. (2005), in Hull and White (2005) or Torresetti et al. (2006b), in Longstaff and Rajan (2007) and in the GPL and GPCL models by Brigo et al. (2007a, 2007b).

**Magnified right tail of the 10 year loss distribution (250 homogenous entities with default probability of 5%), one-factor Gaussian copula assuming 0% recovery with correlation equal to 20% (left) and 90% (right).**




*Source: Torresetti and Pallavicini (2007)*

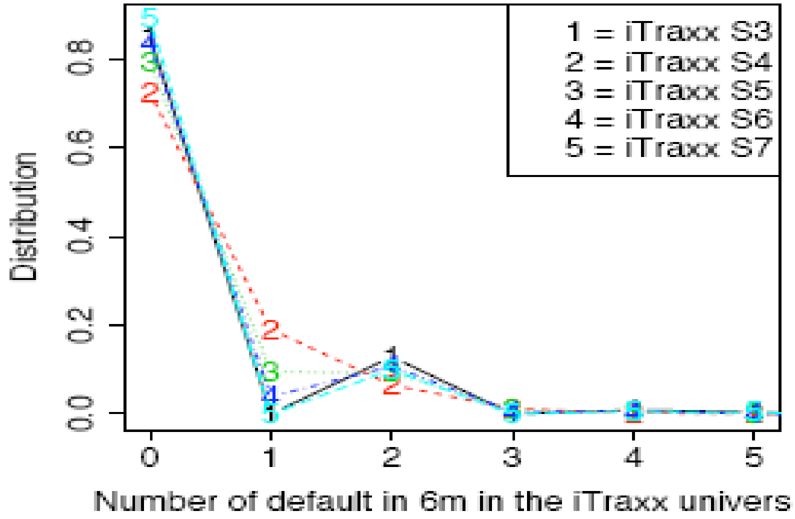
# CPDO Rating Criteria: Pool Losses – GPCL

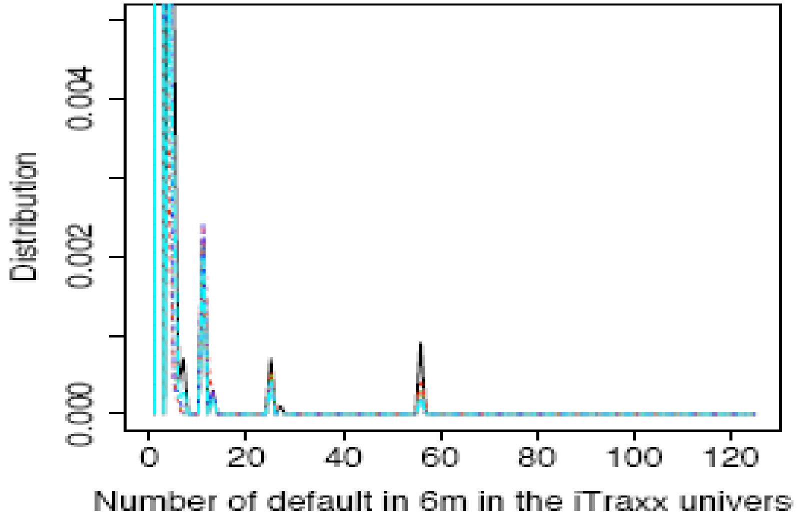
- Under the Generalized Poisson Cluster Loss model the dynamics of the pool loss process L(t) and of the default counting process C(t) are:

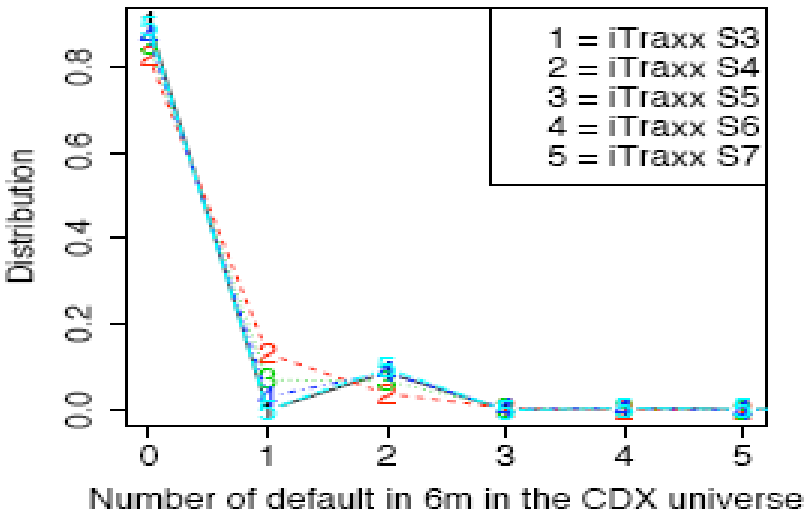
$$dL_t = (1-R)dC_t\,, \quad C_t = \sum_{j=1}^{M} j\, Z_j(t)\,, \quad dZ_j(t) \sim \text{Poisson}\left(\binom{M-C_{t^-}}{j}\tilde{\lambda}_j(t)dt\right)$$

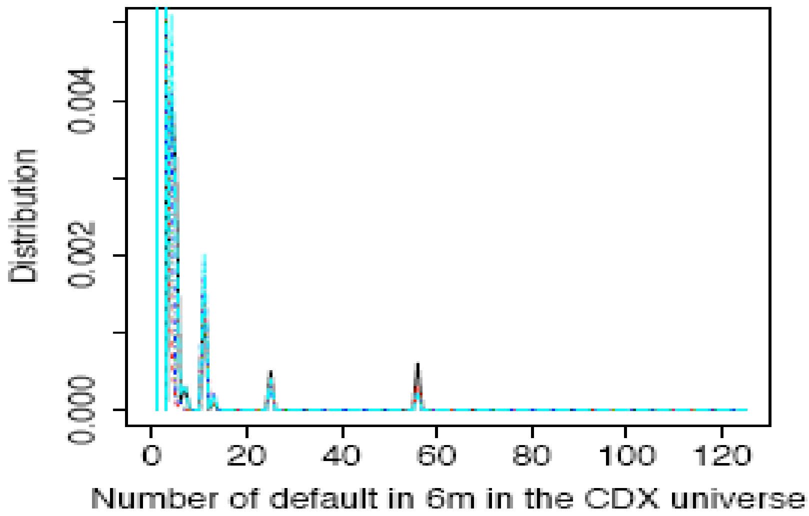
- We fixed the jump sizes in order to evenly space the log default rate from 1=125 to 125=125. Then, for all series we have arbitrarily choose that the intensities $\tilde{\lambda}_j$ with $j \notin J := \{1, 2, 5, 11, 25, 56, 125\}$ are set to zero, namely the counting process can jump only with an amplitude which is listed in ***J***.
- For all weekly market quotes when each series is on-the-run (24 dates = 6 months times 4 weeks per month) we calibrate.
  - The associated intensities $\tilde{\lambda}_j$ are considered to be piecewise-constant functions of tranche time-to-maturity ***t - s*** and depending on calibration time ***s*** only via a common multiplicative factor $\varphi(s)$ allowed to change daily: $\tilde{\lambda}_j(t;s) = \varphi(s)\psi_j(t-s)$
- Once the GPCL model is calibrated to all the cross-sectional data for the same series, all default counting process intensities $\tilde{\lambda}_j$ are rescaled in order to match the six-month probability of default of the underlying pool of names: $1-\exp(-0.5\cdot 0.22\%)$
- Thus we calibrate 120 market quotes (5 tranches times 6 months times 4 weeks) with 30 parameters (7 intensities plus 23 scaling factors)

**Risk Neutral (top plots) versus Objective (bottom plots) distribution of the number of default in 6 months for the iTraxx and CDX pools computed with the GPCL intensities calibrated to the 5y iTraxx tranches**




*Source: Torresetti and Pallavicini (2007)*

# CPDO Rating Criteria: Pool Losses – GPCL

- We calibrated 120 market quotes (5 tranches times 6 months times 4 weeks) with 30 parameters (7 intensities plus 23 scaling factors)

**Average Absolute Standardized Mispricings for the iTraxx and CDX series.The averages are calculated on all weekly market data when the series where on-the-run.**

| series | Roll Date | 0%-3% | 3%-6% | 6%-9% | 9%-12% | 12%-22% |
|---|---|---|---|---|---|---|
| iTraxx S1 | 20-Mar-2004 | 1.7 | 0.8 | 1.9 | 1.1 | 0.8 |
| iTraxx S2 | 20-Sep-2004 | 0.4 | 0.5 | 0.8 | 2.0 | 1.1 |
| iTraxx S3 | 20-Mar-2005 | 5.2 | 6.0 | 1.9 | 1.4 | 1.1 |
| iTraxx S4 | 20-Sep-2005 | 1.6 | 1.6 | 1.5 | 0.8 | 0.9 |
| iTraxx S5 | 20-Mar-2006 | 3.0 | 3.2 | 1.6 | 1.4 | 0.6 |
| iTraxx S6 | 20-Sep-2006 | 1.3 | 1.9 | 1.9 | 1.8 | 0.6 |
| iTraxx S7 | 20-Mar-2007 | 3.2 | 3.1 | 3.6 | 3.8 | 1.5 |
| series | Roll Date | 0%-3% | 3%-7% | 7%-10% | 10%-15% | 15%-30% |
| CDX S2 | 20-Mar-2004 | 0.8 | 0.6 | 1.5 | 1.3 | 0.5 |
| CDX S3 | 20-Sep-2004 | 1.6 | 1.2 | 2.3 | 2.4 | 0.7 |
| CDX S4 | 20-Mar-2005 | 4.7 | 4.1 | 1.7 | 1.6 | 1.9 |
| CDX S5 | 20-Sep-2005 | 1.0 | 0.7 | 1.6 | 1.1 | 2.3 |
| CDX S6 | 20-Mar-2006 | 1.0 | 0.7 | 2.3 | 1.6 | 2.0 |
| CDX S7 | 20-Sep-2006 | 1.4 | 1.2 | 2.6 | 2.6 | 2.7 |
| CDX S8 | 20-Mar-2007 | 2.0 | 1.4 | 1.9 | 4.6 | 5.0 |

*Source: Torresetti and Pallavicini (2007)*

# Results: CPDO default rates

**Probability of default for structured Finance deals of Standard and Poor.**

| | AAA | AA+ | AA | AA- | A+ | A | A- | BBB+ | BBB | BBB- |
|---|---|---|---|---|---|---|---|---|---|---|
| 10Y PD | 0.7% | 1.0% | 1.5% | 1.9% | 2.3% | 2.7% | 3.6% | 4.8% | 7.1% | 12.3% |

*Source: www.standardandpoors.com*

**CPDO average default rate (left table) and rating (right table) under the various assumptions in terms of Roll Down Benefit (RDB) and Mean reversion parameters governing the dynamic of the index spread.**

| | Copula | | GPCL | |
|---|---|---|---|---|
| | RDB=3% | RDB=0% | RDB=3% | RDB=0% |
| Mean Parameter Set $\alpha$=0.55,$\theta$=0.80%,$\sigma$=40.0% | 1.12% | 3.52% | 2.04% | 7.16% |
| Stressed Parameter Set $\alpha$=0.49,$\theta$=1.05%,$\sigma$=38.7% | 2.24% | 9.76% | 4.08% | 11.32% |

| | Copula | | GPCL | |
|---|---|---|---|---|
| | RDB=3% | RDB=0% | RDB=3% | RDB=0% |
| Mean Parameter Set $\alpha$=0.55,$\theta$=0.80%,$\sigma$=40.0% | AA | A- | A+ | BBB- |
| Stressed Parameter Set $\alpha$=0.49,$\theta$=1.05%,$\sigma$=38.7% | A+ | BBB- | BBB+ | BBB- |

*Source: Torresetti and Pallavicini (2007)*

# CPDO Rating Criteria

- In summary we will simulate the CPDO net asset value $V_t$ assuming
  - Deterministic interest rates (**A**)
  - The Credit Index Spread (**B**) dynamic to be an exponential vasicek with parameters estimated historically
  - The Index pool losses (**C**) are modeled with a multifactor gaussian copula or alternatively with a GPCL: in both cases the expected 6 month pool loss is given by the assumed constant rating composition
  - A drift for the mark to market of the protection (**D**) sold $dNPV_t^{T_i^R}$ that represents the roll-down benefit

$$dV_t = \overset{\textbf{A}}{V_t r_t dt - (F_t} + I_t)dt - \overset{\textbf{C}}{\beta_t dL_t} + \left( \overset{\textbf{B}}{S_{T_R^i} \beta_{T_R^i} + \int_{T_R^i}^{t} S_s d\beta_s} \right) dt - X_t dt + \overset{\textbf{D}}{dNPV_t^{T_R^i}}$$

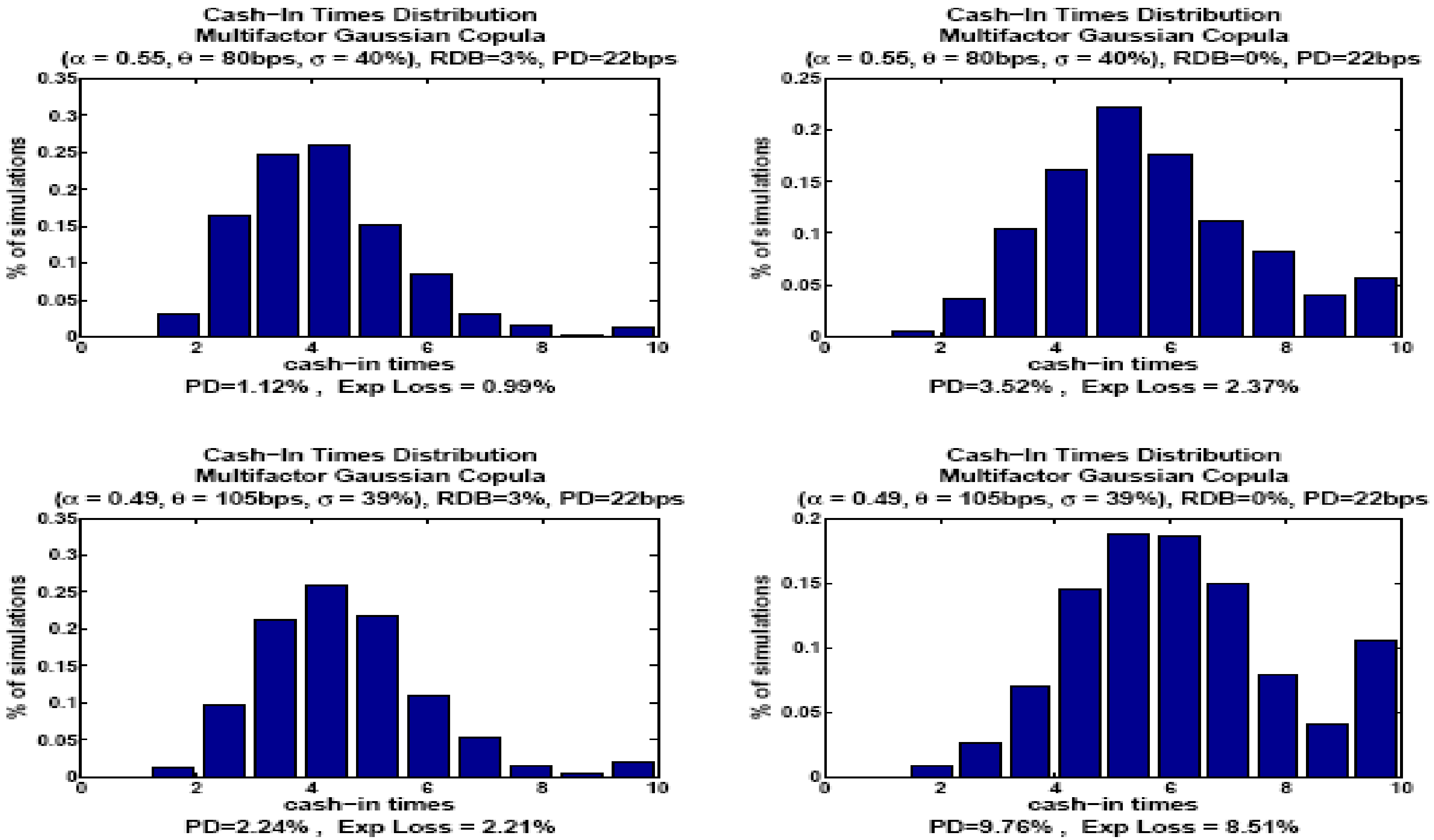
# CPDO Rating Criteria

- The CPDO strategy will be able to reach the bond ceiling before maturity (will not default) if:
    - it will accrue enough leveraged credit default premium (credit spread accrual net of loss payments): **B** – **C**
    - Without triggering the cash-out: the widening in $S_t$ will result in a mark to market loss
    - In case the roll-down benefit (**D**) is assumed positive than with time the structure will also accrue this drift
        - Note that modeling only the 5 year bucket (one factor) we are not admitting any possible curve inversion when recognizing a positive roll-down benefit

# Results: CPDO cash-in times, Gaussian Copula

- The increased riskiness moving away from the Base Criteria can be appreciated also inspecting the cash-in times of the CPDO structure.

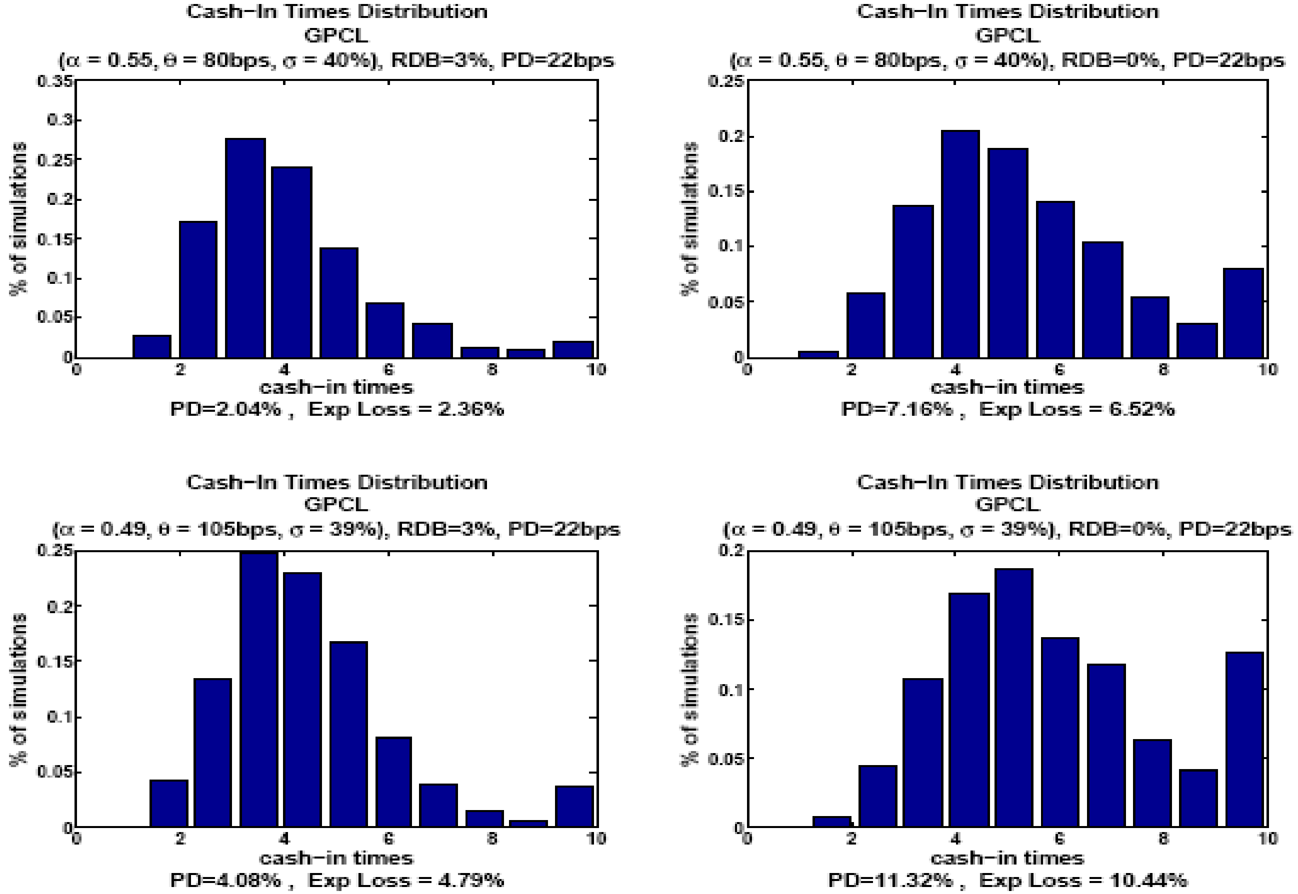
**Distribution in the cash-in times under the multi-factor Gaussian copula approach.**




*Source: Torresetti and Pallavicini (2007)*

# Results: CPDO cash-in times, GPCL

**Distribution in the cash-in times under the multi-factor GPCL approach.**




*Source: Torresetti and Pallavicini (2007)*

# Results: CPDO gap risk provisioning

- Usually a CPDO strategy pays a substantial gap risk fee: 3.5bps per leverage per annum.
  - In case the leverage goes to its maximum, 15, since then on the structure would pay 52.5 bps per annum, 15 times 3.5 bps, as a gap-risk fee.
- The fair gap risk fee in the objective measure is computed dividing the expected gap protection payments, the absolute value of the average NAV when negative or 0 otherwise, by the average leverage to the earliest between maturity, cash-in or cash-out.

**Average gap risk fee in bps per unit of leverage**

| | Copula | | GPCL | |
|---|---|---|---|---|
| | RDB=3% | RDB=0% | RDB=3% | RDB=0% |
| Mean Parameter Set $\alpha$=0.55,$\theta$=0.80%,$\sigma$=40.0% | 0.02 bps | 0.08 bps | 4.32 bps | 1.63 bps |
| Stressed Parameter Set $\alpha$=0.49,$\theta$=1.05%,$\sigma$=38.7% | 0.12 bps | 0.35 bps | 2.51 bps | 2.08 bps |

**Average leverage when the CPDO structure suffers a gap**

| | Copula | | GPCL | |
|---|---|---|---|---|
| | RDB=3% | RDB=0% | RDB=3% | RDB=0% |
| Mean Parameter Set $\alpha$=0.55,$\theta$=0.80%,$\sigma$=40.0% | 5.45 | 5.18 | 11.30 | 13.2 |
| Stressed Parameter Set $\alpha$=0.49,$\theta$=1.05%,$\sigma$=38.7% | 3.8 | 4.5 | 7.9 | 7.37 |

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