DATA-DRIVEN CHARACTERIZATION OF POROUS MATERIALS BY USING FREQUENCY-DEPENDENT MEASUREMENTS

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ABSTRACT

The identification of the acoustical properties of materials plays a key role in the numerical prediction of the acoustic performance and the structural behavior of any mechanical system. Therefore, to obtain an accurate frequency-dependent response, it is essential a suitable choice of the parametric models for each material. However, such models could be inadequate by the material nature or not valid for a given frequency range. In this work, a novel numerical optimization strategy for the characterization of material acoustic properties is introduced, which only uses a reduced number of experimental measurements with the advantage of avoiding parametric model choices.

RESUMEN

La identificación de las propiedades acústicas de los materiales juega un papel clave en la predicción numérica del rendimiento acústico y el comportamiento estructural de cualquier sistema mecánico. Por lo tanto, para obtener una respuesta exacta dependiente de la frecuencia, es esencial una elección adecuada de los modelos paramétricos para cada material. Sin embargo, tales modelos podrían ser inadecuados por la naturaleza del material o no válidos para un rango de frecuencias dado. En este trabajo se introduce una nueva estrategia de optimización numérica para la caracterización de las propiedades acústicas de materiales, que utiliza un número reducido de mediciones experimentales con la ventaja de evitar la elección de modelos paramétricos.

1. INTRODUCTION

Porous materials are widely used for noise mitigation in a large number of acoustic engineering applications in buildings and environment. In this context, it is of great interest to predict the sound absorption performance of these materials when being part of a noise control device (e.g. noise barrier, isolation wall…). Usually, the porous material is modeled as an equivalent fluid having complex acoustic properties, namely, characteristic impedance and wave number. These properties can be easily determined from the intrinsic parameters of the material (e.g. flow resistivity, porosity…) using a theoretical model, remaining acoustical properties (i.e. surface impedance and sound absorption coefficient) being then calculated from the primer. Although many parametric models exist in the literature [1-3, 5, 6], the incessant development of new materials poses the need for alternative predictive tools or fitting methods.

A common procedure to this purpose consists in using an optimization algorithm to fit the intrinsic model parameters that minimize the difference between the measured acoustic properties and those predicted ones using any of the above models. In order to simplify this adjustment, the sound absorption coefficient is normally used in most cases. The main drawback is that the use of a single property may not be enough, or the choice of the reference model could be not suitable for the accurate description of a specific material.
A novel numerical optimization strategy for evaluating the acoustic properties of porous materials is herein proposed. This data-driven methodology uses traditional two-microphone impedance tube setup data to estimate these properties. Three different strategies were tested, each of them successively overcoming some of the limitations of the alternative fitting procedures used for the same purpose (e.g. it does not require to choose a particular model). Results are compared to those obtained using the two-cavity method proposed by Utsuno et al. [9]. Good agreement was found between the measured and predicted acoustic properties. Given that this procedure does not depend on the physical nature of the material itself, it was proven to be more generic than the conventional predictive parametric approaches and could be extended to any type of porous material (i.e. fibrous, granular…).

2. ACOUSTIC CHARACTERIZATION OF PORUS MATERIALS

2.1 Acoustic Properties of Porous Materials

Theory regarding propagation of sound in porous media can be found in [1]. Basically, porous materials attenuate sound mainly due to viscous friction and thermal conductivity in their pore network. If the pore size is small compared to the wavelength of an impinging sound wave, the air inside a layer of porous material can be replaced by an equivalent fluid. The acoustic properties, namely characteristic impedance, Z_C, and wave number, k, that describe this equivalent fluid are then given by

$$Z_C = \sqrt{\rho K}, \tag{1}$$

$$k = \omega\sqrt{\rho/K}, \tag{2}$$

where ρ and K are the complex-valued and frequency-dependent dynamic mass density and dynamic bulk modulus of the saturating fluid, respectively, and ω is the angular frequency. The surface impedance at normal incidence Z_S, for a layer of this equivalent fluid backed by a rigid wall can be calculated from

$$Z_S = -jZ_C \cot(kd), \tag{3}$$

where d is the thickness of the equivalent fluid layer. The sound absorption coefficient, α, can then be obtained from

$$\alpha = 1 - \left|\frac{Z_S-\rho_0 c_0}{Z_S+\rho_0 c_0}\right|^2, \tag{4}$$

where ρ_0 and c_0 are the mass density and sound propagation velocity in air, respectively. A brief description of the methods used to measure experimentally the above properties is given in the following section.

2.2 Experimental Characterization

Experimental methods frequently used to determine the acoustic properties of porous materials use an impedance tube setup [4, 7-9]. Most of them rely on the transfer function method, which uses the pressure data from two microphone positions in a tube in which plane waves propagate. Particularly, Utsuno et al. [9] combined this transfer function method with the two cavity method to estimate the characteristic impedance and wave number of a layer of porous material. Their procedure used the surface impedance measured for a hard backed and an air cavity backed configuration to obtain the intrinsic properties of the porous material. Fig. 1 shows a sketch of this latter impedance tube setup. The characteristic impedance and wave number of a porous sample can thus be deduced from

$$Z_C = \pm\left(\frac{Z_0 Z_0'(Z_1 - Z_1') - Z_1 Z_1'(Z_0 - Z_0')}{(Z_1 - Z_1') - (Z_0 - Z_0')}\right)^{1/2}, \tag{5}$$

$$k = \frac{1}{j2d}\ln\left(\left(\frac{Z_0 + Z_C}{Z_0 - Z_C}\right)\left(\frac{Z_1 - Z_C}{Z_1 + Z_C}\right)\right), \tag{6}$$

where Z_0 and Z_0' are the measured surface impedances for the hard backed and air cavity backed configurations, respectively, Z_1 and Z_1' being the impedances of the corresponding backing layer, which can be deduced from Eq. (3) using the characteristic impedance and wave number of air.

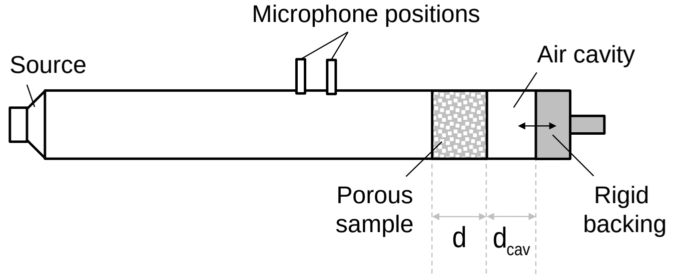

Figure 1. Impedance tube setup proposed by *Utsuno et al.* [9] to determine the characteristic impedance and wave number of porous materials.

2.3 Prediction Parametric Models: Delany-Bazley model for fibrous materials

A common alternative to the previous experimental methods is the use of predictive parametric models to determine the acoustic properties of a given porous material. Many authors have proposed impedance models [2-3, 6], to predict the acoustic behaviour of porous materials when they are used as sound absorbers (e.g. a porous layer backed by a rigid wall). Among these ones, Delany and Bazley [5] developed a model to estimate the acoustic properties of fibrous materials from measurements over a wide frequency range. According to these authors, the characteristic impedance and the wave number of a fibrous material can be determined from

$$Z_C = \rho_0 c_0 (1 + 0.0571 X^{-0.754} - j0.087 X^{-0.732}), \tag{7}$$

$$k = \frac{\omega}{c_0}(1 + 0.0978 X^{-0.700} - j0.189 X^{-0.595}), \tag{8}$$

where $X = \rho_0 f/\sigma$, being f the frequency and σ the flow resistivity of the material. Hence, the acoustic behaviour of a fibrous material may be described as a function of its flow resistivity using these simple empirical parametric relations.

3. DATA DRIVEN APPROACH

The main concern for practitioners consists in ensuring an adequate choice of the parametric porous model. This most accurate selection is not always possible to be known *a priori*, since it depends on the acoustic nature of the material samples. In fact, an inadequate model selection could ruins any parameter model fitting. As a partial remedy of these drawbacks, in the present work the proposed data-driven methodology avoids the choice model, since it does not require to impose any functional dependency on the parameters in terms of the frequency and it is only based on the experimental measurements. More precisely, the proposed approach uses intensively the numerical solution of an inverse problem, which fits a discrete set of frequency-dependent experimental measurements such as the surface impedance, described as follows: considering a vector $\vec{p}(\omega)$, where each component is the value of a relevant acoustic quantity (real or imaginary parts of characteristic impedances, dynamic mass density,…), these quantities are straightforwardly computed by means of the solution of a fitting problem at each fixed frequency value ω:

$$\vec{p}(\omega) = \arg\min_{\vec{p}} \left(\frac{|Z(\omega) - Z_{\mathrm{s,anl}}(\omega, \vec{p})|^2}{|Z(\omega)|^2} \right), \tag{9}$$

where $Z_{\mathrm{s,anl}}(\omega, \vec{p})$ is the surface impedance of a porous layer computed numerically by determining the acoustic propagation of plane waves through the multilayer media using the Transfer Matrix Method (TMM). Additionally, this fitting problem ensures the correct sign for all the acoustic quantities involved in the vector $\vec{p}(\omega)$. In this approach, it is essential to select conveniently the acoustic unknowns which are part of the quantity vector $\vec{p}(\omega)$, and consequently, the acoustic quantities which will be assumed as primal unknowns in the inverse problem.

Firstly, it is necessary to take into account that if the real and the imaginary part of the mass density ρ and the bulk modulus K are considered as the primal unknowns naively, the fitting relative error is almost negligible but spurious oscillations distort the parameter frequency-response due to the exponential dependency of the TMM matrix coefficients with respect to these acoustic quantities. In order to mitigate this situation, instead of using the dynamic mass density and the dynamic bulk modulus as primal unknowns, the fitting problem (9) has been rewritten replacing the real and imaginary part of the bulk modulus by a novel pair of unknowns: $\delta = \mathrm{Re}(k)d$ and $M = e^{\mathrm{Im}(k)d}$, which involves the wave number of the porous material k and its thickness d.

In this work, three different strategies have been tested to deal with the ill-posedness of the inverse problem. In all of them, the numerical value of the surface impedance $Z_{\mathrm{s,anl}}(\omega, M, \delta, \mathrm{Re}(\rho), \mathrm{Im}(\rho))$ has been computed considering δ, M, the real and the imaginary part of the dynamic mass density of the material as primal unknowns. Thereby, in the first fitting problem only the measurements of a porous layer backed by a rigid wall have been used. This fitting problem is ill-posed since different parameter vectors lead to the same value of the surface impedance (clearly, from Eq. (3), it can be checked that there exist an infinite number of different pair of values for the wave number and the characteristic impedance, which lead to the same surface impedance value).

In the second fitting problem not only the surface impedance measurements of a porous layer backed by a rigid wall but also the measurements of a porous layer backed by an air gap terminated in a rigid wall have been used. Despite considering two data sets, the problem remains ill-posed, since parameter δ is well-determined except for an integer multiple of π (notice that δ drives the complex phase of some TMM matrix coefficients related to the porous layer). To avoid this multiplicity of solutions in the inverse problem, in the third fitting problem the surface impedance measurements of the setup with and without air gap are considered, and additionally, to fix the value of quantity δ, the low-frequency limit of the real part of the dynamic mass density is assumed known (see [7] for a further discussion about a consistent definition of these limits for fibrous limp materials). In this manner, the fitting problem is well-posed and in the case of porous material samples consisting in a unique layer, the numerical results obtained with the proposed methodology are equivalent to those ones reported with the Utsuno approach [9], once both low-frequency limits are imposed equal in the two techniques.

4. RESULTS AND DISCUSSION

Some experimental data have been used for performing the numerical simulations shown in this section. These ones have been measured by using the procedure explained in [9]. In the legends of the plots they are labelled as “Utsuno”. In the first fitting problem, the experimental data used are the measurements in the setting without air gap. The numerical result of the fitting is shown in Figures 2 and 3. The relative errors resulting from this fitting are around 10^{-6}%.

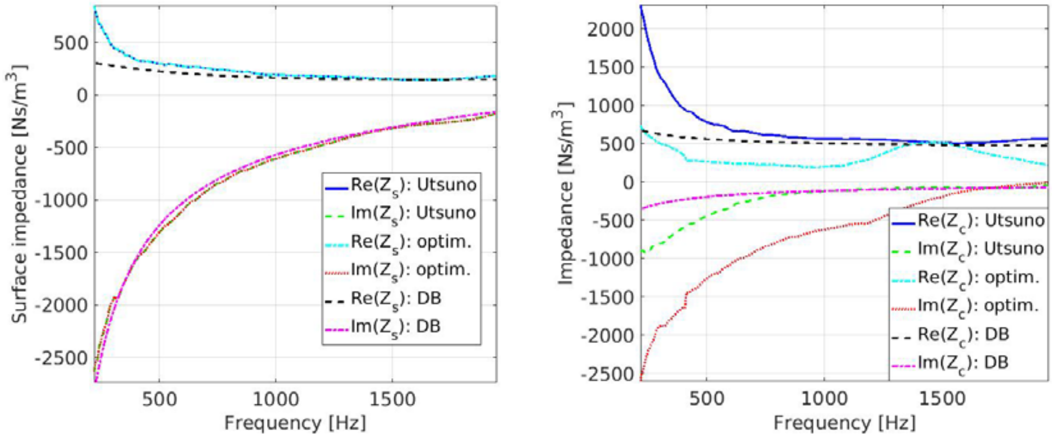

Figure 2. Real and imaginary part of the surface impedance (left plot) and real and imaginary part of the characteristic impedance (right plot) computed with the fitting values, taking into account the setting without air gap, in comparison with the Utsuno estimations and with the fitting of the Delany-Bazley model (DB).

The values of the characteristic impedance, the dynamic mass density and the wave number of the material computed by using the optimal values obtained with the fitting procedure are shown in the right plot of Figure 2 and in the plots of Figure 3, respectively.

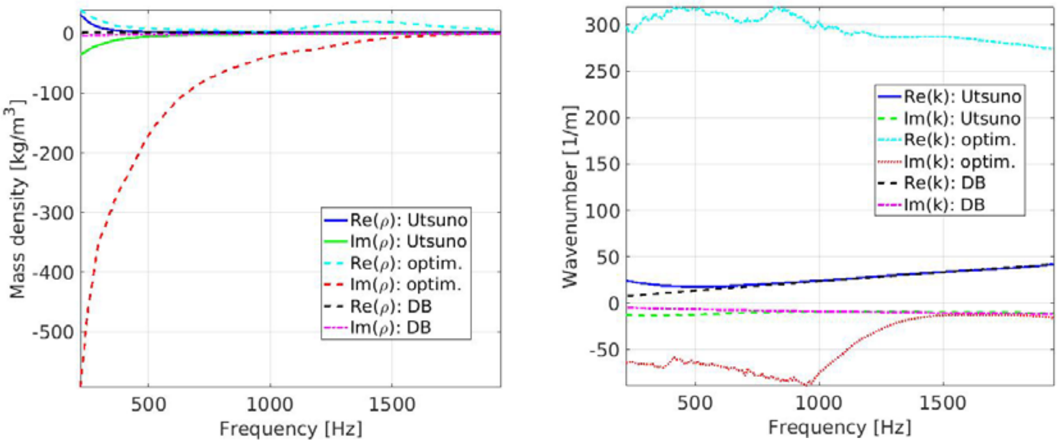

Figure 3. Real and imaginary part of the dynamic mass density (left plot) and real and imaginary part of the wave number (right plot) computed with the fitting values, taking into account the setting without air gap, in comparison with the Utsuno estimations and with the fitting of the Delany-Bazley model (DB).

Although the relative error reached in the fitting of the surface impedance is small, the characteristic impedance, the dynamic mass density and the wave number are different from those ones computed using the Utsuno procedure. The reason of this disagreement is the ill-posedness of the inverse problem using only a set of surface impedance measurements. In order to solve this drawback, a new inverse problem has been solved by using two sets of measurements: a set without air gap and also other one considering the measurements in the setup with air gap in the Kundt's tube. The numerical result of the fitting is shown in Figure 4. In this case, the relative errors resulting from this fitting are around $10^{-6}\%$.

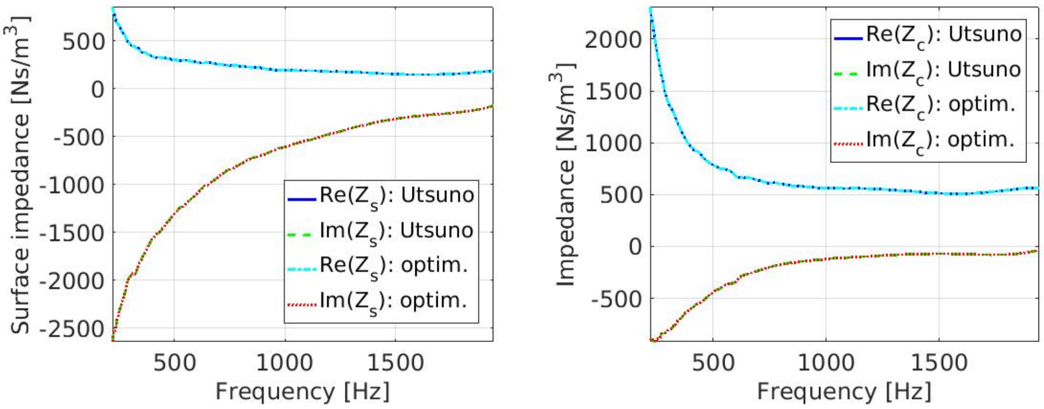

Figure 4. Real and imaginary part of the surface impedance (left plot) and real and imaginary part of the characteristic impedance (right plot) computed with the fitting values, taking into account the setting without and with air gap, in comparison with respect to the Utsuno estimations.

The values of the characteristic impedance, the dynamic mass density and the wave number of the material computed by using the optimal values, obtained with the fitting procedure, are shown in the right plot of Figure 4 and the plots of Figure 5, respectively.

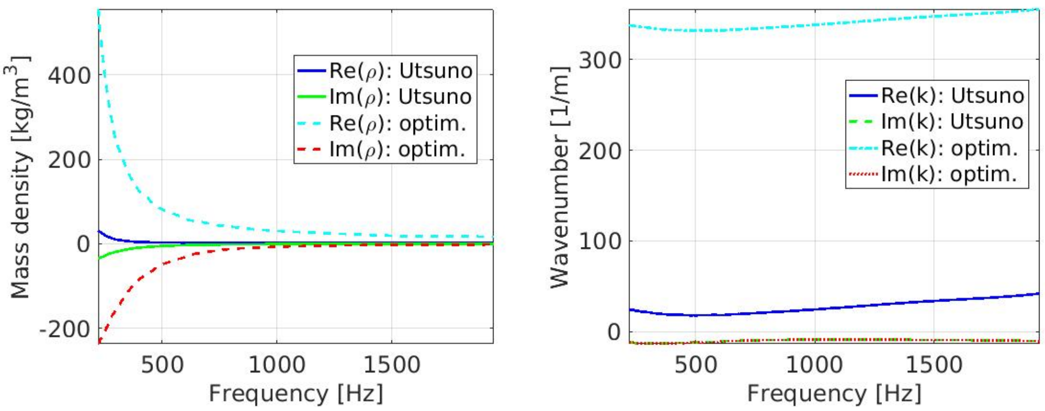

Figure 5. Real and imaginary part of the dynamic mass density (left plot) and real and imaginary part of the wave number (right plot) computed with the fitting values, taking into account the setting without and with air gap, in comparison with respect to the Utsuno estimations.

Again, the relative error in the fitting problem is small and in this case, as it can be observed in the right plot of Figure 4, the results for the computation of the characteristic impedance are accurate in comparison with the Utsuno estimations. However, Figure 5 shows that the dynamic mass density and the wave number reach different values from those ones reported with the Utsuno technique. Once more, this disagreement is produced by the ill-posedness of the fitting problem (now only affecting to the quantity δ). To deal with this mild problem due to the multiple solutions of the inverse problem, in addition to the experimental frequency-dependent measurements used in both settings (without and with air gap), the low-frequency limit of the real part of the dynamic mass density has been assumed known *a priori*, this is, taking into account the experimental measurements, it has been fixed $\lim_{\omega\to 0}\mathrm{Re}(\rho) = 41.5\mathrm{kg/m^3}$ in order to compare accurately the fitting numerical results with those ones obtained with the Utsuno technique (the value of this limit has been computed by extrapolation on the experimental measurements to obtain an identical low frequency behavior as in the Utsuno prediction). The fitting numerical results on the surface impedance are shown in the left plot of Figure 6. In this case, the relative errors resulting from this fitting are again around 10^{-6}%.

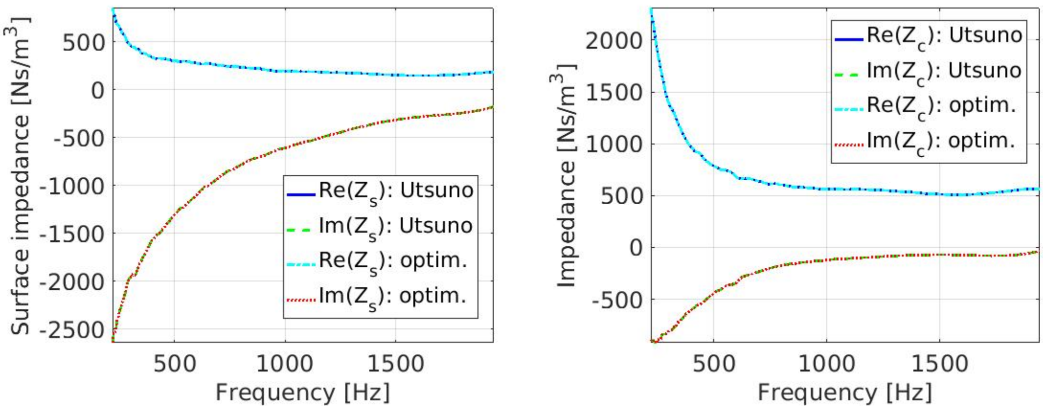

Figure 6. Real and imaginary part of the surface impedance (left plot) and real and imaginary part of the characteristic impedance (right plot) computed with the fitting values, taking into account both settings and considering known the low-frequency data for the dynamic mass density in comparison with respect to the Utsuno estimations.

The values of the characteristic impedance, the dynamic mass density and the wave number of the material computed by using the optimal values, obtained with the fitting with both settings and considering known the low-frequency limit of the real part of the dynamic mass density are shown in the right plot of Figure 6 and Figure 7, respectively.

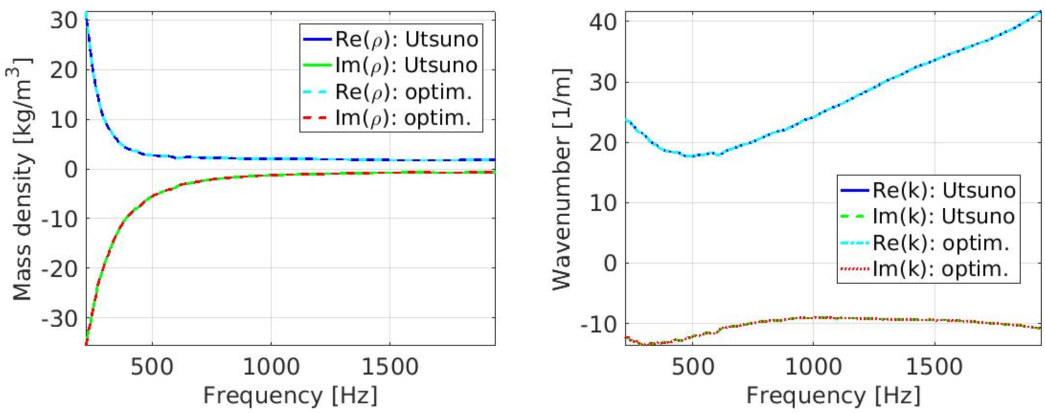

Figure 7. Real and imaginary part of the dynamic mass density (left plot) and real and imaginary part of the wave number (right plot) computed with the fitting values, taking into account both settings and considering known the low-frequency data for the dynamic mass density, in comparison with respect to the Utsuno estimations.

Considering the assumptions mentioned above, the relative errors obtained in the fitting procedure are small and the frequency response of the characteristic impedance, the dynamic mass density and the wave number of the sampled material coincide with those ones obtained from using the Utsuno technique. Notice that the value of the low-frequency limit of the real part of the dynamic mass density has been chosen in all the setting cases to be equal to that one reported in the Utsuno results.

5. CONCLUSIONS

A data-driven approach has been proposed to predict the acoustical properties of porous materials. The adopted procedure is based on a fitting problem at each frequency value, but without the need of any theoretical parametric model compared to earlier works by taking advantage of the experimental data obtained using a two-microphone impedance tube setup. The developed method has been validated by means of experimental data obtained using the two-cavity method worked out by Utsuno et al. [9], resulting in relative errors around $10^{-6}\%$ in all cases. It has been shown that a single parameter optimization may lead to misleading results. However, a more rigorous fitting procedure like the one herein proposed using to sets of experimental measurements (with and without air gap) have been shown necessary for a proper description of the porous material. Furthermore, the variation in the predicted fitting values when using one or more input properties has been assessed, and served to confirm the previous assertion. In summary, it was shown that the acoustic properties of porous materials can be accurately predicted by adopting this data-driven approach. Even though further research must be carried out for other type of materials, the application of this procedure to derive the properties of porous materials, films or micro-perforated plates commonly used in more complex multilayer devices are of great interest.

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