How to Share Railways Infrastructure Costs?[1]

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Abstract: In this paper we propose an infrastructure access tariff in a cost allocation problem arising from the reorganization of the railway sector in Europe. To that aim we introduce the class of infrastructure cost games. A game in this class is a sum of airport games and what we call maintenance cost games, and models the infrastructure costs (building and maintenance) produced when a set of different types of trains belonging to several agents makes use of a certain infrastructure. We study some properties of infrastructure cost games and provide a formula for the Shapley value of a game in this class. The access tariff we propose is based on the Shapley value of infrastructure cost games.
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1 Introduction

In this paper we deal with a cost allocation problem arising from the reorganization of the railway sector in Europe, after the application of the EEC directive 440/91 and the EC directives 18/95 and 19/95, which involve the separation between infrastructure management and transport operations. In this situation two main economic problems arise. One is to allocate the track capacity among the various operators. This issue has been treated, for instance, in Nilsson (1995), Brewer and Plott (1996) and Bassanini and Nastasi (1997). The second problem is to determine the access tariff that the railway transport operators must pay to the firm in charge of the infrastructure management for a particular journey. This tariff should take into account several aspects such as the a priori profitability and social utility of the journey, congestion issues, the number of passengers and/or goods transported, the services required by the operator, infrastructure costs, etc. The tariff is conceived in an additive way, i.e. as the sum of various tariffs corresponding to the various aspects to be considered.

The main motivation of this paper is a practical one. We were approached by *Ferrovie dello Stato*[2] (the italian national railway company) to study how the infrastructure costs should be allocated to the operators through a fair *infrastructure access tariff* (i.e. we were asked to define one part of the additive access tariff: that corresponding to the infrastructure costs). In this work we treat this problem from a game theoretical point of view, making use of the Shapley value. The Shapley value is a very important solution concept for TU-games, which has excellent properties and has been applied successfully in cost allocation problems (see Shapley (1953), Tijs and Driessen (1986), Young (1994) and Moulin and Shenker (1996)). Moreover, in our particular problem, it is especially appropriate because of the following two reasons.

1. It is well-known that the Shapley value is an additive solution. This feature fits well with the "additive nature" of the access tariff, as commented above.

2. In this paper we will show that the infrastructure access tariff based on the Shapley value can be computed very easily (using, once more, the additivity of the Shapley value). In a practical environment this is certainly an important property. Take into account that a very big

[2]Ferrovie dello Stato is the coordinator of the EuROPE-TRIP research project, sponsored by the European Community. Formally, our research has been requested and financed by the European Community.

amount of fees will have to be computed by the infrastructure manager every new season, so computational issues become highly relevant.

Let us now describe informally the problem we are facing. Consider a railway path (for instance, Milano-Roma), that is used by different types of trains belonging to several operators, and consider the problem of dividing among these trains the infrastructure costs. Clearly it is a problem of joint cost allocation. To settle the question, one can see the infrastructure as consisting of some kinds of "facilities" (track, signalling system, stations, etc.). Different groups of trains need these facilities at different levels: for example, fast trains need a more sophisticated track and signalling system, compared to local trains, for which instead station services are more important (particularly in small stations).

So, a straightforward approach can be that of viewing the infrastructure as a "sum" of different facilities, each of them required by the trains at a different level of cost.

Furthermore, infrastructure costs can be seen as the sum of "building" costs and "maintenance" costs (for a better understanding of the distinction between these two types of costs, we refer to the example in section 4). If we consider only building costs, especially in the case of a single facility, we are facing a problem similar to the so-called "airport game" (see, for instance, Littlechild and Owen (1973) and Dubey (1982)). For what concerns maintenance costs, it seems to be a reasonable first order approximation to assume that they are proportional both to the building costs and to the number of trains that use the facility.

Similar considerations extend to related problems: for example the costs for a bridge, to be used by small and big cars. There are building costs, that are different in the case of a bridge for small or big cars, and maintenance costs, that can be assumed to be proportional to the number of vehicles using the bridge, and to the kind of bridge needed.

In this paper we analyze these infrastructure cost games (sums for various facilities of a building cost game and a maintenance cost game) from the point of view of the Shapley value. In section 2 we introduce and briefly study the infrastructure cost games. In section 3 we provide a simple expression of the Shapley value for this class of games. In section 4 we elaborate an example where we apply the models and results presented in sections 2 and 3.

2 Infrastructure Cost Games

For simplicity, we concentrate first on infrastructure cost games when we are dealing with the building and maintenance of one facility. To begin with, we recall the definition of an "airport game".

Definition 2.1 Suppose we are given k groups of players $g_1, \ldots, g_k$ with $n_1, \ldots, n_k$ players respectively and k non-negative numbers $b_1, \ldots, b_k$. The *airport game* corresponding to $g_1, \ldots, g_k$ and $b_1, \ldots, b_k$ is the cooperative (cost) game $< N, c >$ with $N = \cup_{i=1}^{k} g_i$ and cost function c defined by

$$c(S) = b_1 + \cdots + b_{j(S)}$$

for every $S \subseteq N$, where $j(S) = \max\{j : S \cap g_j \neq \emptyset\}$.

Airport games are cost games for the building of one facility (for instance, a landing strip) where the wishes of the coalitions are linearly ordered. Coalitions desiring a more sophisticated facility (a larger landing strip) have to pay at least as much as coalitions desiring a less sophisticated facility (a smaller landing strip). Every b_i represents the extra building cost that should be made in order that a facility that can be used by players in groups $g_1, ...g_{i-1}$ can also be used by the more sophisticated players in group g_i. Airport games are known to be concave. Consequently, the Shapley value of such a game provides a core element. Sometimes we will refer to an airport game as a *building cost game*. Denote by $B(g_1, ..., g_k)$ the set of all building cost games with groups of players $g_1, ..., g_k$.

In airport games costs for the building of one facility are modeled. Now we consider the maintenance costs of this facility, which lead to the class of "maintenance cost games". Basic assumptions are that maintenance costs are increasing with the degree of sophistication of the facility and that maintenance costs are proportional to the number of users.

Definition 2.2 Suppose we are given k groups of players $g_1, \ldots, g_k$ with $n_1, \ldots, n_k$ players respectively and $k(k+1)/2$ non-negative numbers $\{\alpha_{ij}\}_{i,j \in \{1,\ldots,k\}, j \geq i}$. The *maintenance cost game* corresponding to $g_1, \ldots, g_k$ and $\{\alpha_{ij}\}_{i,j \in \{1,\ldots,k\}, j \geq i}$ is the cooperative (cost) game $< N, c >$ with $N = \cup_{i=1}^{k} g_i$ and cost function c defined by

$$c(S) = \sum_{i=1}^{j(S)} |S \cap g_i| A_{ij(S)} \tag{1}$$

for every $S \subseteq N$, where $A_{ij} = \alpha_{ii} + ... + \alpha_{ij}$ for all $i, j \in \{1, ..., k\}$ with $j \geq i$.

The interpretation of the numbers α_{ij} and A_{ij} is the following. Suppose that one player in g_i has used the facility. In order to restore the facility up to level i (the level of sophistication desired by this player) the maintenance costs are $A_{ii} = \alpha_{ii}$. If, however, the facility is going to be restored up to level $i+1$, then extra maintenance costs α_{ii+1} will be made. So, in order to restore the facility up to level j (with $j \geq i$) the maintenance costs are $A_{ij} = \alpha_{ii} + \alpha_{ii+1} + ... + \alpha_{ij}$. Hence, $c(S)$ represents the maintenance costs corresponding to the facility up to the level $j(S)$ (so that all the players in S can use it), after all players in S have used it. Observe that, for every $i \leq j$, the more sophisticated the facility is (the larger j is), the higher the maintenance costs produced by a player in g_i are. In Section 4 we provide an example which illustrates the above definition of a maintenance cost game.

We denote by $M(g_1, ..., g_k)$ the set of all maintenance cost games with groups of players $g_1, ..., g_k$. Obviously, to characterize a game $< N, c > \in M(g_1, ..., g_k)$ it is equivalent to give either the set of parameters $\{\alpha_{ij}\}_{i,j \in \{1,...,k\}, j \geq i}$ or the set of parameters $\{A_{ij}\}_{i,j \in \{1,...,k\}, j \geq i}$.

The following decomposition of a maintenance cost game $< N, c > \in M(g_1, ..., g_k)$ will be useful. For every $S \subseteq N$,

$$\begin{aligned} c(S) &= \sum_{i=1}^{j(S)} |S \cap g_i| A_{ij(S)} = \\ &= \sum_{i=1}^{j(S)} |S \cap g_i| (\alpha_{ii} + ... + \alpha_{ij(S)}) = \sum_{i=1}^{k} \sum_{j=i}^{k} \alpha_{ij} c^{ij}(S), \end{aligned}$$

where

$$c^{ij}(S) = \begin{cases} |S \cap g_i| & \text{if } j \leq j(S) \\ 0 & \text{if } j > j(S) \end{cases}$$

for all $i, j \in \{1, ..., k\}$ with $j \geq i$.

We know that building cost games are concave. The following result shows that this is not true for maintenance cost games. Moreover, it shows that maintenance cost games are essentially neither concave nor balanced.

Theorem 2.1 *Let $< N, c >$ be the maintenance cost game corresponding to $g_1, \ldots, g_k$ and $\{\alpha_{ij}\}_{i,j \in \{1,...,k\}, j \geq i}$. Then the following four statements are equivalent:*

(1) $< N, c >$ is concave

(2) $< N, c >$ is balanced

(3) $\sum_{i\in N} c(i) \geq c(N)$

(4) $\alpha_{ij} = 0$ *for every* $j > i$.

Proof. The implications (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are clear. For the implication (3) $\Rightarrow$ (4) suppose that (3) holds. Then

$$\sum_{i=1}^{k}\sum_{j=i}^{k}\alpha_{ij}n_i = c(N) \leq \sum_{i\in N} c(i) = \sum_{i=1}^{k}\alpha_{ii}n_i$$

which implies that $\alpha_{ij} = 0$ for every $j > i$. For the implication (4) $\Rightarrow$ (1) suppose that (4) holds. Note that c^{ii} defined as above is an additive characteristic function for every $i \in \{1, ..., k\}$. Hence, c can be expressed as a non-negative combination of additive characteristic functions. Thus, $< N, c >$ is concave. ■

Now we can introduce the class of infrastructure cost games.

Definition 2.3 A *one facility infrastructure cost game* with groups of players $g_1, \ldots, g_k$ is the cooperative (cost) game $< N, c >$ with $N = \cup_{i=1}^{k} g_i$ and cost function $c = c_b + c_m$ such that $< N, c_b > \in B(g_1, \ldots, g_k)$ and $< N, c_m > \in M(g_1, \ldots, g_k)$. An *infrastructure cost game* with groups of players $g_1, \ldots, g_k$ is the cooperative (cost) game $< N, c >$ with $N = \cup_{i=1}^{k} g_i$ and cost function $c = c^1 + ... + c^l$ such that, for every $r \in \{1, ..., l\}$, $< N, c^r >$ is a one facility infrastructure cost game with groups of players $g_{\pi^r(1)}, \ldots, g_{\pi^r(k)}$, where π^r is a permutation of $\{1, ..., k\}$.

From the definition above we see that a one facility infrastructure cost game is the sum of a building cost game plus a maintenance cost game with the same groups of players ordered in the same way. An infrastructure cost game is the sum of a finite set of one facility infrastructure cost games with the same groups of players but, perhaps, ordered in a different way. This means that group i can require a higher level of sophistication than group j for facility r, whereas group j requires a higher level of sophistication than group i for facility s. Because of this reason, it is not true that every infrastructure cost game is a one facility infrastructure cost game. An interesting consequence of Theorem 2.1, the concavity of airport games and the additivity of the Shapley value is the following. Since an infrastructure cost game is the sum of building cost games and maintenance cost games, then its Shapley value is the sum of allocations, which are moreover core allocations for those such games having a non-empty core.

The class of infrastructure cost games is the model we designed to solve the practical problem which motivates this work: how to allocate in a fair way the infrastructure costs to the users of a certain railway path. A game in our class describes the infrastructure costs imputable to every possible collection of users. Now we have to choose an allocation rule which allocates the total cost to the users. As we announced in the introduction of this paper, we chose the Shapley value because of the two reasons already discussed. The access tariff we propose for a certain path in a certain time period is simply the Shapley value of the infrastructure cost game corresponding to this path and time period.

Note that an infrastructure cost game is the sum of a finite collection of airport games and maintenance cost games. It is well known that there is a simple expression of the Shapley value for airport games (see Littlechild and Owen, 1973). In the next section we obtain a simple expression of the Shapley value for maintenance cost games. Hence, since the Shapley value is additive, we can compute easily the Shapley value of an infrastructure cost game even when the number of players is large, which will be the case in practice: take into account that the players here are the trains using the path in a certain period. Thus, we are proposing an access tariff system which is at the same time reasonable (based on a general theory of fairness) and computable in an efficient way.

3 The Shapley Value of a Maintenance Cost Game

This section contains a theorem providing a simple expression of the Shapley value of a maintenance cost game.

Theorem 3.1 *Let $< N, c >$ be the maintenance cost game corresponding to the groups $g_1, ..., g_k$ (with $n_1, ..., n_k$ players respectively) and to $\{\alpha_{lm}\}_{l,m \in \{1,...,k\}, m \geq l}$. Then, for every $i \in N$,*

$$\begin{aligned} \varphi_i(c) &= \alpha_{j(i)j(i)} + \sum_{m=j(i)+1}^{k} \alpha_{j(i)m} \frac{n_m + ... + n_k}{n_m + ... + n_k + 1} \\ &+ \sum_{m=2}^{j(i)} \sum_{l=1}^{m-1} \alpha_{lm} \frac{n_l}{(n_m + ... + n_k)(n_m + ... + n_k + 1)}, \end{aligned}$$

where $\varphi_i(c)$ denotes the i-th component of the Shapley value of the game $< N, c >$ and $j(i)$ is the group to which i belongs (i.e. $i \in g_{j(i)}$).

Proof. Recall that $c = \sum_{l=1}^{k} \sum_{m=l}^{k} \alpha_{lm} c^{lm}$ where

$$c^{lm}(S) = \begin{cases} |S \cap g_l| & \text{if } m \leq j(S) \\ 0 & \text{if } m > j(S). \end{cases}$$

Then, since the Shapley value is linear,

$$\varphi_i(c) = \sum_{l=1}^{k} \sum_{m=l}^{k} \alpha_{lm} \varphi_i(c^{lm})$$

for all $i \in N$. It is clear that, for every $l \in \{1, ..., k\}$, c^{ll} is an additive characteristic function and that

$$\varphi_i(c^{ll}) = \begin{cases} 1 & \text{if } i \in g_l \\ 0 & \text{in any other case.} \end{cases} \tag{2}$$

Suppose now that $l < m$. In this case only players in $g_l \cup (\cup_{r=m}^{k} g_r)$ are not null players. By symmetry we may put $\varphi_i(c^{lm}) = a$ for every $i \in g_l$ and $\varphi_i(c^{lm}) = b$ for every $i \in \cup_{r=m}^{k} g_r$. In order to compute a take $i \in g_l$ and note that for every $S \subseteq N \backslash \{i\}$ we have

$$c^{lm}(S \cup \{i\}) - c^{lm}(S) = \begin{cases} 0 & \text{if } j(S) < m \\ 1 & \text{else.} \end{cases}$$

So, if the players of N are ordered at random, a is the probability that player i has at least one predecessor in $\cup_{r=m}^{k} g_r$. Equivalently, if the players of N are ordered at random, a is the probability that player i is not the first player of the players in $\{i\} \cup (\cup_{r=m}^{k} g_r)$. Consequently,

$$a = \frac{n_m + ... + n_k}{n_m + ... + n_k + 1}. \tag{3}$$

Thus, by symmetry and efficiency,

$$b = \frac{n_l - n_l a}{n_m + ... + n_k} = \frac{n_l}{(n_m + ... + n_k)(n_m + ... + n_k + 1)}. \tag{4}$$

Now, in view of (2), (3) and (4) the proof is concluded. ■

As we mentioned above, the Shapley value of the corresponding infrastructure game is our proposal to share railways infrastructure costs. It is clear that, using Theorem 3.1 and the formula for the Shapley value of an airport game, the computations that should be made are not difficult; however, the potentially very large amount of data that will have to be handled to compute a very large collection of fees makes necessary to have a good computer program to do it. For this purpose, we have prepared a software package that will be delivered to *Ferrovie dello Stato*, the coordinator of EuROPE-TRIP. The name of this package is ShRInC (Sharing Railways Infrastructure Costs). It has been created with the collaboration of Luisa Carpente and Claudia Viale.

Obviously, from a game theoretical point of view, there are many interesting questions concerning infrastructure cost games that have not been treated here. The main motivation of this paper is to report the practical solution we proposed for the real problem of allocating railways infrastructure costs. In Norde et al (1999), we study other game theoretical properties of infrastructure cost games.

4 An Example

In this section we illustrate our solution with an example. We shall elaborate it on data taken from Baumgartner (1997). The aim of that paper is to provide "order of magnitude" of costs concerning the railway system: we shall exploit it to analyze a rough but realistic example. In practical models, making a realistic example uses to be an enlightening exercise. Here, for instance, the example we are proposing shows that our building or maintenance cost games do not necessarily correspond to real building or maintenance costs. Actually, the costs for one facility can be decomposed into:

- a fixed part (in the sense that it does not depend on the number of players), that corresponds to the building cost game associated with this facility, and
- a variable part (in the sense that it is proportional to the number of players), that corresponds to the maintenance cost game part.

For simplicity, we shall concentrate on a single element (the track), even if Baumgartner (1997) provides data also for other elements (line, catenary, signalling and security system, etc.), that can be analyzed in a similar fashion. If we consider one kilometer of track, from Baumgartner (1997) we get two kind of costs[3], that depend on the type of train (slow/fast) and on the number of trains running. More precisely, we have both *renewal costs* and *repairing costs*. According to this division of costs we will divide the track into two facilities: "track renewal" and "track repairing".

[3]We assumed the weight of 50Kg for a meter of rail and made a linear approximation of the costs given in table 2 of Baumgartner (1997).

Renewal costs can be approximated by the following formula:

$$RWC = 0.001125X + 11,250$$

where RWC are the renewal costs per kilometer and per year (expressed in swiss francs) and X measures the "number" of trains, expressed in yearly TGCK (TGCK means Tons Gross and Complete per Kilometer).

So, if we assume for ease of exposition that all of the trains running are of the same weight, the facility "track renewal" has a fixed component (to be included in our building costs), and a part which is proportional to the number of trains running (to be included in our maintenance costs). If the assumption of equal weight cannot be sustained, our model still fits: simply divide trains into groups of similar weight. In such a case each group will have different unitary maintenance costs.

Similarly, for the facility "track repairing", costs can be given by analogous formulas:

$$RPC_s = 0.001X + 10,000$$

$$RPC_f = 0.00125X + 12,500.$$

RPC_s denotes the repairing costs (in swiss francs) per kilometer and per year of a track prepared only for slow trains, whereas RPC_f denotes the repairing costs (in swiss francs) per kilometer and per year of a track prepared for all trains. X denotes the same as before.

So, consider one kilometer of line, which will be used this year by a total weight of 10^7 TGCK (corresponding to 20,000 trains, assuming a weight per train of approximately 500 tons). Assume that 5,000 trains are fast and that the remaining are slow. The infrastructure cost game that can be used to allocate the costs is $< N, c >$ given by:

- $N = g_1 \cup g_2$, g_1 being the set of slow trains ($n_1 =$ 15,000) and g_2 being the set of fast trains ($n_2 =$ 5,000).
- $c = c^1 + c^2$, c^1 and c^2 being one facility infrastructure cost games both having the same groups of players and ordered in the same way: g_1, g_2.

Now, c^1 and c^2 are characterized by the following parameters.

- $c^1 : b_1^1 = 11,250$; $b_2^1 = 0$; $\alpha_{11}^1 = 0.5625$; $\alpha_{12}^1 = 0$; $\alpha_{22}^1 = 0.5625$.
- $c^2 : b_1^2 = 10,000$; $b_2^2 = 2,500$; $\alpha_{11}^2 = 0.5$; $\alpha_{12}^2 = 0.125$; $\alpha_{22}^2 = 0.625$.

Hence, making use of Theorem 3.1 and the formula for the Shapley value of an airport game, it is easy to check that, if $\varphi_s(c)$ and $\varphi_f(c)$ denote the Shapley value of a slow and a fast train respectively, then:

- $\varphi_s(c) = \frac{b_1^1}{n_1+n_2} + \alpha_{11}^1 + \frac{b_1^2}{n_1+n_2} + \alpha_{11}^2 + \alpha_{12}^2 \frac{n_2}{n_2+1} = 2.25$
- $\varphi_f(c) = \frac{b_1^1}{n_1+n_2} + \alpha_{22}^1 + \frac{b_1^2}{n_1+n_2} + \frac{b_2^2}{n_2} + \alpha_{22}^2 + \alpha_{12}^2 \frac{n_1}{n_2(n_2+1)} = 2.75.$

These are the fees, in swiss francs, that every slow and fast train (respectively) should pay per kilometer of track used, according to our solution. Clearly, in front of a specific allocation problem regarding a specific line, with specific transport operators and trains, appropriate data should be collected. Here, we only presented an illustrative approximation to a real example.

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