# SOME FLUID-STRUCTURE INTERACTION PROBLEMS IN LUBRICATED SYSTEMS

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**Abstract**
We consider a lubricated mechanism consisting of two non-parallel surfaces in hydrodynamic contact. The bottom surface, assumed planar and horizontal, moves with a constant horizontal translation velocity. The wedge between the two surfaces is filled with an incompressible fluid. This wedge is assumed to satisfy the thin-film hypothesis, so that the pressure does not depend on the vertical coordinate and obeys the Reynolds equation which reads, in non-dimensional form (and assuming time independence):

$$\begin{cases} \nabla \cdot \left[h^3(x)\nabla p\right] = \frac{\partial h}{\partial x_1} & x=(x_1,x_2)\in\Omega \\ p=0 & x\in\partial\Omega \end{cases} \tag{1}$$

where $h$ is the non-dimensional distance between the surfaces and $\Omega$ the two-dimensional region in which proximity takes place.

We now suppose that the forme of the upper surface (which is given by $h$) is not completely known. We suppose that a vertical force $F$ is applied on the upper body by means of an articulated joint parallel to $x_2$, at a given position $x_1^0$. The form of $h$ is then given up to two degrees of freedom, the vertical displacement $a$ and the tilt (or pitch) angle $\theta$. We then have

$$h(x) = h_0(x) + a + \theta\, x_1 \tag{2}$$

where $h_0$ is a given function that accounts for a reference shape of the upper surface. The displacement $a$ and the angle $\theta$ result of the equilibrium of forces and moments acting on the upper rigid body. We then have

$$\int_\Omega p\,dx = F \tag{3}$$

$$\int_\Omega p(x_1 - x_1^0)\,dx = 0 \tag{4}$$

The main goal of this work is to prove the existence of a solution $(p,a,\theta)\in H_0^1(\Omega)\times I\!R\times I\!R$ of the coupled system of equations (1), (2), (3) and (4).

This problem can be viewed as an interaction fluid-structure problem, where the classical Navier-Stokes equations are replaced by a Reynolds equation, due to the thin film hypothesis.

We also consider the case when cavitation can occurs in the fluid film, in which case the Reynolds equation (1) must be replaced by the corresponding Reynolds variational inequality which is written

$$\text{Find } p\in K \text{ such that } \quad \int_\Omega h^3\nabla p\cdot(\varphi - p) \geq \int_\Omega h\frac{\partial}{\partial x_1}(\varphi - p) \quad \forall\varphi\in K$$

where $K=\{\varphi\in H_0^1(\Omega),\ \varphi\geq 0\}$.