Inverse problem for a non coercive Lubrication model

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One of the crucial problem in the design of the surfaces for lubricated devices is to meet the imposed load by the designers. This kind of problem appears in many applications like journal bearing, mechanical face seals, hard disk drive and so on. The mathematical models lead to optimal control problems with the Reynolds equation as state equation. However these problems present some mathematical and numerical difficulties when the cavitation, the well-known phenomenon in fluid mechanic when fluid is no longer homogeneous and takes diphasic aspect with the appearance of air bubbles, is considered. Taking cavitation into account implies to consider a new nonlinear operator as state-equation. In this talk we first recall some previous studies dealing with different model of cavitation and then we present a model in which the cavitation is taken into account by the slightly compressible Elrod model . This model has the advantage to conserve the mass flow and to take into consideration, implicitly, the free boundary phenomenon and compressibility. So an optimal control problem where the state is solution of a nonlinear degenerate partial differential equation is considered. This problem presents many mathematical difficulties essentially bound to the non coerciveness of the state equation and the complexity of the considered objective function. Furthermore the thickness can be discontinuous and so we are lead to consider a more general functional context which is the space of the functions of bounded variation. To deal with these difficulties a double regularization is introduced, while approaching the degenerate part of the state equation by a particular sequence of monotonous and continuously differentiable functions and in turn to formulate a sequence of regularized control problems.

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