Asymptotic thin films. Viscoelastic models.

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The purpose of this work is to provide a rigourous justification of the validity of a model of a thin film flow for viscoelastic fluids described by a law of the Oldroyd type.
In many mechanical works, the authors obtained in a heuristic way various models of viscoelastic fluids in thin films, for the generalized Maxwell model [4] or for Phan Thien-Tanner laws [1]. In this work, a mathematical approach is developed for the Oldroyd-B model.
Viscoelastic fluids are non-newtonian fluids whose constitutive equation (relation between the stress tensor and the shearing rate) is a differential law. By coupling this law with the Navier-Stokes equations, one obtains the following macroscopic model:

$$\begin{cases} \partial_t U + U \cdot \nabla U - \eta(1-r)\Delta U + \nabla p - \operatorname{div} \sigma = 0, \\ \operatorname{div} U = 0, \\ \lambda \left(\partial_t \sigma + U \cdot \nabla \sigma + g(\nabla U, \sigma) \right) + \sigma = 2r\eta D(U). \end{cases}$$

This model couples equations on the velocity U of the fluid, its pressure p and the stress tensor σ. The existence of a global solution to this problem has been proved in [3].
Supposing the domain to be thin (one dimension is small with respect to the two others), it was shown in [2] that one obtains formally a system coupling only the velocity $U^* = (u^*, w^*) \in \mathbb{R}^2 \times \mathbb{R}$ and the pressure p^*, for which it is possible to show the existence of a weak solution:

$$\begin{cases} -\eta(1-r)\partial_z^2 u^* - \partial_z \beta^* + \nabla_x p^* = 0, \qquad \text{with} \quad \beta^* = \dfrac{\eta r \partial_z u^*}{1 + K^2 |\partial_z u^*|^2}, \\ \partial_z p^* = 0, \\ \operatorname{div}_x u^* + \partial_z w^* = 0. \end{cases}$$

We will show the convergence of the first system towards this limit system by writing an asymptotic expansion for each unknown. Under a realistic assumption on the parameters of the system, one can prove the convergence of the asymptotic expansions thanks to energy estimates on the remainders.

References

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