# On magnetic recording

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**Abstract.** We consider a limit case of a system of two equations arising in magnetic recording for a one-dimensional domain. The system models the tape deflection when it is driven over a magnetic head profile. The unknowns of the problem are the position of the tape "$u$" and the pressure of the air "$p$" in the gap between the tape and the head. The position of the tape is modeled by the beam equation and the pressure (when the tape is close to the head) satisfies the compressible Reynolds equation.

After nondimensionalization one obtains the following system of equations:

$$\frac{\partial(ph)}{\partial x} - \epsilon \frac{\partial}{\partial x}\left(\alpha h^2 \frac{\partial p}{\partial x} + \beta h^3 p \frac{\partial p}{\partial x}\right) = 0, \qquad x_i < x < x_i + L_i,$$

$$-\frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^4 u}{\partial x^4} = k(p-1)\left(\sum_{i=1}^{n} \chi_{[x_i, x_i+L_i]}\right), \qquad 0 < x < L,$$

$$u(x) = h(x) + \delta(x), \qquad h(x) > 0 \text{ if } x_i \le x \le x_i + L_i, \quad \text{for } 1 \le i \le n$$

where

$$0 < x_i < x_i + L_i < x_{i+1} < x_{i+1} + L_i < L, \qquad \text{for } 1 \le i \le n-1,$$

$\chi_{[x_i, x_i+L_i]}$ is the characteristic function of the interval $[x_i, x_i + L_i]$ and typically

$$\alpha \sim \frac{1}{10},\ \beta \sim 1,\ x_n + L_n - x_1 \sim 1,\ L \sim 10,\ k \sim 10^4,\ \epsilon \sim 10^{-2},\ \mu \sim 10^{-3};$$

$x_i$ lies near of the middle of the interval $(0, L)$ for $1 \le i \le n$.

We first study the case $\epsilon = \mu = 0$, then the system is reduced to a second order nonlinear equation, where the unknown $u$ appears evaluated in a finite set of distinguished points $\{x_i\}_{i=1}^n$ of the domain.

$$-\frac{\partial^2 u}{\partial x^2} = k \sum_{i=1}^{n} \left(\frac{u(x_i) - \delta(x_i)}{u(x) - \delta(x)} - 1\right) \chi_{[x_i, x_i+L_i]}, \qquad 0 < x < L.$$

Results for small and positive $\epsilon$ and $\mu$ will be also presented.